HIERARCHICAL BAYESIAN ESTIMATION OF HETEROGENEOUS DYNAMIC PANEL DATA MODEL

BY

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ABSTRACT

The estimation of static panel data model assumes homoscedastic error terms that is often violated in most economic models and when this happens, the Dynamic Panel Data Model (DPDM) is specified. The DPDM presumes correlation between lagged dependent variable and individual (unit) specific effects, resulting to heterogeneity among the units. The parameters of DPDM are usually estimated using classical approach which has no control for heterogeneity of the error terms leading to non-consistent estimates of the parameters. This study was aimed at deriving Hierarchical Bayesian Estimator (HBE) capable of handling Heterogeneous Dynamic Panel Data Model (HDPDM).

The HDPDM, $y_{it} = \delta_i y_{i,t-1} + \beta_{0i} + X_{1it}\beta_{1i} + X_{2it}\beta_{2i} + \varepsilon_{it}$, was generalised as $y_{it} = X^*_{it}\gamma_i + \varepsilon_{it}$ for $X^*_{it} = (X_{it} y_{i,t-1})'$ and $\gamma_i = (\beta_i'\delta_i)'$; where i indicates that the marginal effect of X^* on y varies across the units, y is $(NT \times 1)$ vector of dependent variable, X^* is $(NT \times \mathrm{NK})$ matrix of unit specific regressors, γ is $(NK \times 1)$ vector of parameters, and ε is $(NT \times 1)$ vector of error terms. The HBE was derived in two stages. First Stage of Hierarchical (FSH) parameter priors were $\gamma_i \mid y, h \sim \mathrm{N}(\mu_\gamma, V_\gamma)$ and $h \sim G(\underline{s}^{-2}, \underline{v})$, where μ_γ and $\underline{s}^{-2}$ are means, V_γ is variance-covariance and $\underline{v}$ is degree of freedom with independent Normal-Gamma prior. Second Stage of Hierarchical (SSH) parameter priors were $\mu_\gamma \sim N(\underline{\mu}_\gamma, \underline{\Sigma}_\gamma)$ and $V_\gamma \sim W(\underline{v}_\gamma, \underline{V}_\gamma^{-1})$, where ($\underline{\mu}_\gamma$ and $\underline{v}_\gamma$) are means and ($\underline{\Sigma}_\gamma$ and $\underline{V}_\gamma^{-1}$) are variance-covariance with independent Normal-Wishart prior. To account for heterogeneity, the FSH was derived from SSH to produce consistent estimates. Data were simulated using Markov chain Monte Carlo approach with $\delta_i \sim B(0,1)$, $\beta_{0i} \sim N(0, 0.25)$, $\beta_{1i} = 2$ and $\beta_{2i} = 3$ to obtain Posterior Estimates (PEs) at 10,000 iterations. Three experiments for the individual (N) and time (T) were considered: $N < \mathrm{T}$ (20, 50), $N = \mathrm{T}$ (50, 50) and $N > \mathrm{T}$ (100, 15). The performance of the HBE was assessed using Numerical Standard Error (NSE). Relatively Non-informative Prior (RNP) (h= 0.04, 0.03, 0.02, 0.01) and Informative Prior (IP) (h= 25, 30, 50, 70) were examined to check for the sensitivity of priors on the PEs.

The derived HBE was $p(\gamma_i, h \mid y) = p(y \mid \gamma_i, h).p(\gamma_i).p(h)$. The PEs of SSH for μ_γ $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.1009, 0.1326, 1.0808, 4.0607, NSE $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.0002, 0.0008, 0.0017, 0.0068 and V_γ was 0.0007 for $N < \mathrm{T}$; μ_γ $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.0154, 0.0061, 3.9674, 1.9943, NSE $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.0002, 0.0005, 0.0014, 0.0041 and V_γ was 0.0001 for $N = \mathrm{T}$ and μ_γ $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.1535, 0.1635, 2.0456, 2.8847, NSE $(\delta, \beta_0, \beta_1, \beta_2)$ were 0.0001, 0.0004, 0.0006, 0.0006 and V_γ was 0.0000 for $N > \mathrm{T}$. The obtained V_γ gave a constant error variance for all the parameters across the units. The $N > \mathrm{T}$ option produced the least NSE, hence outperformed the other two. The PEs of FSH for $\gamma_i = (\delta_i, \beta_{0i}, \beta_{1i}, \beta_{2i})$, $i = 1, 2, ..., 5$ were δ_i= 0.1425, 0.1443, 0.1501, 0.1275, 0.1333. β_{0i}= 1.0172, 0.9123, 0.8553, 1.0172, 0.2225, β_{1i} = 1.5539, 1.5911, 1.5761, 1.5539, 1.4245, β_{2i}= 2.5193, 2.5345, 2.5005, 2.5193, 2.5231. These reflected the marginal effects of X^* on y across the units. The RNP with values of h for β_1 = 1.5400, 1.5404, 1.5413, 1.5431 and β_2 = 2.5358, 2.5336, 2.5354, 2.5358, while for IP, β_1= 1.5418, 1.5399, 1.5427, 1.5420 and β_2= 2.6358, 2.6336, 2.6369, 2.6373. The estimated parameters with changes in h values were closely identical to the pre-set β_1 and β_2 values. Thus, indicating the sensitivity of prior information on the PEs.

The Hierarchical Bayesian Estimator facilitated by suitable prior information solved the problem of heterogeneity in the dynamic panel data model. Therefore, will find useful applications in panel data economic models.

Keywords: Heterogeneous effect, Relatively non-informative prior, Lagged dependent variable, Normal-Gamma prior, Normal-Wishart prior.

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Dedication

This thesis is dedicated to God almighty, the author and the finisher of my faith,
my husband, Oluwafemi T. Akinlade
And my children;
Ayomikun, Ayomipo and Ayomipe Akinlade

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My profound gratitude goes to God Almighty, the One who was, who is and who remains good forever in my life. The author and the finisher of my faith, you are highly exalted.

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Certification

I certify that this work was carried out by Mrs Akinlade, YemisiOmolara in the Department of Statistics, Faculty of Science, University of Ibadan, Nigeria.

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Some acronyms used in the thesis

Acronyms	Full Meaning
ABGMM	Arellano-Bond Generalised Method of Moment
AH	Anderson- Hsiao
BC	Bootstrap-based Correction procedure
BLUE	Best Linear Unbiased Estimator
BTW	Between Estimator
CCE	Common Correlated Effects
CCEMG	Common Correlated Effects Mean Group
CS	Cross-Sectional
DPD	Dynamic Panel Data
FD	First-Differenced
FD-GMM	Difference Generalized Methods of Moments estimator
FE	Fixed Effects
FEE	Fixed Effects Estimator
FGLS-SUR	Fixed Generalised Least Squares Seemingly Unrelated Regression
GLS	Generalised Least Squares
GMG	General Mean Group
GMM	Generalised Method of Moments
GQMLE	Gaussian Quasi Maximum Likelihood Estimator
IEE	Interactive-Effects Estimator
IV	Instrumental Variables
J-IV	Just-identified Instrumental Variable estimator
LD-GMM	Long-Difference GMM estimator
LIML	Limited Information Maximum Likelihood
LM	Lagrangian Multilpier
LSDV	Least Squares Dummy Variable
MC	Monte Carlo
MCMC	Markov Chain Monte Carlo
MG	Mean Group
ML	Maximum Likelihood
NLS	National Longitudinal Survey
OLS	Ordinary Least Squares
OLS-FE	Ordinary Least Squares Fixed Effects
POLS	Pooled Least Squares
PSID	Panel Study of Income Dynamic
RCPD	Random-Coefficients Panel Data
RCR	Random Coefficient Regression
REML	Random Effect Maximum Likelihood
SE	Shrinkage Estimators
SYS-GMM	System Generalized Method of Moment estimator

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Introduction

The linear regression model with panel data is a repeated cross-sections on statistical units over a given period of time. It is the pooling of observations of cross-sectional data such as households, countries, firms and so on over several time periods, Baltagi(2008). Statistics survey deals with cross-sectional data reporting each of many different units at a single point in time while time series data are used to describe a single entity, usually an economy, consumer price indices or market. Panel data have become increasingly useable and popular in both developing and developed nations, for examples, two of the popular panel datasets in the United States of America are the Panel Study of Income Dynamic (PSID) and the National Longitudinal Survey (NLS) of labour market experience, Baltagi(2005) and Hsiao (2003). Panel data models in macroeconomics are commonly increased in the past years with the maximum accessibility of cross country datasets which enable researchers to check common information about the model parameters across all units. The analyses and policy evaluations increasingly require taking observations of cross-sectional data over several time periods existing across sectors, markets and countries into account.

In contemporary econometric methodology, panel data analysis plays a vital role in that it takes benefit of the grouping structure to address essential economic questions completely than it is possible with single kinds of data. Specifically, the grouping structure (panel data) can be used to analyse models with complicated forms of heterogeneity across individuals. Also, panel data ensure that the methods used to estimate parameters are suitable to the problem at hand. Under normal circumstances, one would expect the computation of panel data estimator to be more complicated than cross-sectional or time series data, but in many sectors and markets data, the availability of panel data simplifies estimation and statistical inference. Panel data suggest that individuals, states, firms or countries are heterogeneous, hence, it is useful to control the individual heterogeneity. With heterogeneity, variations about the model parameters across the individual are studied. Empirical analysis of panel data balance heterogeneity with some statistically meaningful concept of information accumulation and commonality of interest.

Panel data give more informative data, less collinearity, degree of freedom, efficiency, and variability among variables. They give information on the time-ordering of cases. They are better able to discover and accese effects that are simply not noticeable in pure time series or pure cross-sectional data, and are more suitable for studying the dynamic of adjustment. Also, panel data allow us to construct and test more complicated behavioural models than purely cross-sectional or time-series data; Panel data also provide a means of resolving the econometric problems that often arise in empirical studies, especially the studies of unobserved individual variables that are correlated with independent variables.

Panel data models can be indicated as a static or dynamic. Static panel data model is the traditional model which assumes that the error terms have constant variance (homoscedastic) through the random individual effect. The general framework for the static panel data analysis is a regression model of the form

$$y_{it} = X_{it}\beta + \varepsilon_{it} \quad (1.1)$$

Where i= 1, 2, . . . ,N and t = 1,2,. . . , T. For N and T are the cross-section and time series dimension respectively. y_{it} is the response variable, β is the parameter to be estimated, X_{it} is explanatory variable and ε_{it} are error terms.

A dynamic panel data model can be described as model with lagged dependent variable on the right-hand side of the equation of a panel data model. Dynamic panel data are exhibiting phenomenal growth and consistently becoming popular among the behavioural and social researchers.One of the advantages of dynamic panel data over cross-sectional data is that dynamic panel data provide sufficient information about earlier time periods for dynamic relationships to be investigated rather than observation of a single point in time. Another advantage of the dynamic panel data is that the totality of time series data containing the possibility that underlying micro-economic dynamics may be unnoticed by aggregation biases, and scope that the panel data offers to examine heterogeneity in adjustment dynamics between different types of individuals, households or firms.

Panel data models allow the researcher to understand the dynamics of adjustment. For example Balestra and Nerlove (1966) on the dynamic demand for natural gas, Baltagi and Levin (1986) on dynamic demand for an addictive commodity like cigarettes, Holtz-Eakin *et al* (1988) on a dynamic wage equation, Arellano and Bond (1991) on a dynamic model of employment, Blundell *et al*. (1992) on a dynamic model of company investment, Islam (1995) on a dynamic model for growth convergence, and Ziliak (1997) on a dynamic lifecycle labour supply model.

Dynamic models are of concern in a full range of economic applications which include Euler equations for family uptake, empirical models of economic growth and adjustment cost models for organisation factor demand, education and democracy, empirical model of economic growth and so on, Bond and Windmeijer (2002). Even when much interest are not placed on regression coefficients of lagged dependent variables, to recover consistent estimates of other parameters, dynamic in the process may be of importance. Many economic processes show dynamic adjustment over time and overlook the dynamic points of the data. This is not only a loss of relevant and vital information, but can result to severe misspecification biases in the parameter estimation. A model with lagged dependent variables for many omitted variables can be controlled to a large extent. Additional complications related to the degree of heterogeneity in the lagged dependent variable parameters may occur when the regression equation includes dynamic terms, pooling the data assuming common autoregressive dynamics across all units is one option.

The model for simple autoregressive order (1) dynamic model is given as,

$$y_{it} = \delta y_{i,t-1} + X'_{it}\beta + (\eta_i + \varepsilon_{it}); \quad |\delta| < 1; \qquad i=1,2,...,N; \qquad t=1,2,...T \qquad (1.2)$$

Where y_{it} is an observation on series for individual (unit) i in time t, $y_{i,t-1}$ is the one-period lagged value of the dependent variable with parameter δ, X'_{it} is the row vector of explanatory variable and β is the coefficient of the explanatory variable. η_i is an unobserved individual-specific time-invariant effect which allows for heterogeneity in the average of the y_{it} series across units, and ε_{it} is the error term. Where these two assumptions $\eta_i \sim \text{IIN}(0, \sigma^2_\eta)$ and $\varepsilon_{it} \sim \text{IIN}(0, \sigma^2_\varepsilon)$ are independent of each other and among themselves. The assumptions hold when the error terms are exogenous across the units. The individual numbers (N) for the available data is assumed to be large along time period numbers (T) which is assumed to be small while the asymptotic properties are considered as individual number (N) becomes large with T fixed. The individual effects are stochastic implying that they are correlated with the lagged dependent variable $y_{i,t-1}$ unless the distribution of the $y_{i,t-1}$ is degenerated with the assumption that error terms are uncorrelated. These jointly indicated that the Ordinary Least Squares (OLS) estimator of δ in equation (1.2) is inconsistent, since the independent variable $y_{i,t-1}$ is positively correlated with disturbance terms due to the presence of the individual effects, Baltagi(2005). Autocorrelation due to the inclusion of a lagged dependent variable among the regressors and individual-specific effects characterizing the heterogeneity among the units leads to certain problems which are dealt with by different estimation methods such as OLS, Fixed Effects Estimator (FEE), Least Squares Dummy Variable (LSDV) among others. Some of the basic problems introduced by the presence of a lagged dependent variable are; since y_{it} is a function of η_i , it follows immediately that $y_{i,t-1}$ is also a function of η_i. Therefore, $y_{i,t-1}$, a right-hand regressor in equation (1.2), is correlated with the error term. This makes the OLS estimator biased and inconsistent even if the ε_{it} are not serially correlated. It is unfortunate that estimation of dynamic panel model is problematic. The problem arises as a consequence of relatively short time series component for the Fixed Effects (FE) specification. Thus,Nickel(1981) instigated the usual Hurwicz type bias into OLS estimation of a FE dynamic panel data model. In the random effect specification, Generalised Least Squares (GLS) estimators are likewise biased due to a correlation between the equations error terms through the lagged variableand the individual effect, Sevestre and Trognon(1985). Instrumental Variables (IV) estimators appear to be the most favourable form of consistent estimation for both fixed and random effect specifications in the classical approach. The LSDV for autoregressive panel data models is inconsistent for small T, Nickel (1981), therefore, in such cases, IV estimators and Generalised Method of Moments (GMM) are both widely used.

More generally, dynamic panel data models with autoregressive coefficients are extensively used in the analysis of economic data, Arellano and Honore(2001).

Pesaran and Smith (1995) examined dynamic panel data models with heterogeneous autoregressive coefficients, centred on models that assume the same autoregressive coefficients for all units but allow intercepts to vary across units. They showed that not accounting for the heterogeneity produces inconsistent estimates of the mean autoregressive coefficient and pooling of observations will result in asymptotic non-consistent estimates. To address this problem, they suggested the mean group estimator which entails the estimation of the regression model unit-by-unit as a way out to this problem. The execusion of this technique requires a T that is greatly large to assure unbiased coefficients in each cross- sectional data, Pesaran and Smith(1995).

In the case of small T, Phillips and Sul (2002) proposed Fixed Generalised Least Squares Seemingly Unrelated Regression (FGLS-SUR) estimator to address both cross-sectional dependence and dynamic heterogeneity but this estimator failed to deal with the problem of cross-sectional dependence. Their estimator is impracticeable when N>T, Pesaran (2006) suggests the Common Correlated Effects Mean Group (CCEMG) estimator. Small sample bias means that this approach is likely not to work when T is small, as in the case of mean group estimator.

More so, Bai (2009) present dynamic panel estimators that are specifically designed for micro panel data and allow for cross-sectional correlation but dynamic heterogeneity is not addressed.

Parameter heterogeneity has been of interest in econometrics for a long time, showing the inherent instability of economic relationships that can arise from consumer tastes, structural change, aggregation problems, or misspecification, Nerlove (1967).

Consider a dimensional dynamic panel model with minimal restrictions on the parameter heterogeneity:

$$y_{it} = \delta_{it} + \beta_i X_{it} + \varepsilon_{it} \tag{1.3}$$

Where

$$\beta_i = \lambda + \lambda_i \tag{1.4}$$

Swamy (1970) consider equations (1.3) and (1.4) and introduced the random coefficients model where λ_i is a random process with $E(\lambda_i) = 0$. A highly related approach was suggested by Pesaran and Smith (1995) and it is called Mean Group (MG) OLS. Regressions are undertaken on each individual to obtain consistent estimates for β_i and these are then averaged to derive a consistent estimate of λ usually as a simple average:

$$\hat{\beta}_{MG} = N^{-1} \sum_{i=1}^{N} \hat{\beta}_i \tag{1.5}$$

Explicit estimates of λ_i can also be of inherent economic interest, for instance when i represents an individual.

In practice, there are two essential econometric problems in estimating dynamic panel data models. Firstly, parameters are known to be biased in models with fixed effects and lagged dependent variables, and secondly the homogeneity assumptions that are often imposed on the coefficients of the lagged dependent variable can lead to critical biases when the dynamics are heterogeneous across the cross-section individuals, Weinhold (1998). An additional problem of injecting dynamics into a panel data model is the potential bias induced by heterogeneity of the cross- section individuals. Pesaran and Smith (1995) explore this problem many times. They show that parameter estimates derived from panel data are inconsistent in dynamic models even for large N and T. Panel data model with large N and small T, assume homogeneity of the slope coefficients when the observations are pooled.

In the modern econometric methodology, panel data analysis plays avital role because it is often possible to take merit of the grouping structure to address important economic questions more totally than is possible with simpler kinds of data. In particular, the grouping structure can be used to estimate models with complicated forms of heterogeneity across units. For panel data studies with large N and small T, itis usual to pool the observations, assuming homogeneity of the slope coefficients. Moreover, with the increasing time dimension of panel datasets, some researchers including Robertson and Symons(1992) and Pesaran and Smith(1995) have questioned the poolability of the data across heterogeneous units. Instead, they argue in favour of heterogeneous estimates that can be combined to obtain homogeneous estimates if the need arises. To buttress this point, Robertson and Symons (1992) studied the properties of some panel data estimators when the regression coefficients vary across individuals, that is, when they are *heterogeneous* but are assumed *homogeneous.* The basic conclusion is that severe biases can occur in dynamic estimation even for relatively small parameter variation, Robertson and Symons (1992) and Tiao and Zellner (1964).

Estimation of panel data models with lagged dependent variables and cross-sectionally dependent errors has been considered by Moon and Weidner (2015), who proposed a Gaussian Quasi Maximum Likelihood Estimator (QMLE). Moon and Weidner analysis assumes homogeneous coefficients, and therefore it is not applicable to dynamic panels with unobserved individual specific effects. Similarly, the Interactive-Effects Estimator (IEE) developed by Bai (2009) also allows for cross-sectionally dependent errors, but assume homogeneous slopes. Song (2013) extends the analysis of Bai(2009) by allowing for a lagged dependent variable as well as coefficient heterogeneity, but provides results on the estimation of cross-section specific coefficients only.

However, working with panel data requires care to ensure that the techniques used are appropriate to the problem at hand. With heterogeneity, an additional unit of data is at best only partially informative about the model parameters common to all units. Bayesian approach is suggested by Hsiao *et al* (1999) to solve dynamic heterogeneity bias for single-equation dynamic panel data models. Bayesian Econometrics seeks the combination of Bayesian statistics in relevant ways to models and phenomena of interest to economists. It is based on few simple rules of probability.

More generally, Bayesian statistics presumes that there are certain parametric distributions for the unknown coefficients. It attaches the probability model of consideration by incorporating prior information regarding the unknown parameters and the likelihood function of the data. It assumes that all quantities, including the parameters, are random variables. Therefore, prior probability distributions are introduced for the parameters. This prior distribution expresses a state of knowledge or ignorance about the parameters before the data are obtained.

The Markov Chain Monte Carlo (MCMC) method is considered in the context of Bayesian inference to simulate a Markov Chain so as to incur posterior samples from the joint posterior distribution of parameters of interest under a certain prior probability density.

Bayesian estimation via repeated sampling from posterior distributions facilitates hierarchical modeling of dynamic panel data, whether of random effects, correlated or unstructured observed error levels or time varying regressor effects. As noted by Davidian and Giltinan (1995), the random effects are treated as parameters in Bayesian MCMC estimation, and are ordinarily not integrated out as often done in classical approaches.

In addition, one of the characteristics of the Bayesian estimation is that no conjugate priors are imposed for the individual-specific effects, that is, the effects are not necessarily required to follow a normal prior distribution to ensure that the posterior distribution fall in the same family class of the prior as with the conjugate prior assumption imposed in regular Bayesian regression model. Rather the posterior distribution form is non-analytical in nature implementing a MCMC algorithm in the Bayesian inference to estimate the model. The models with large number of parameters to be estimated require additional effort of Bayesian technique called hierarchical Bayesian technique.

Hierarchical Bayesian estimation is a multi-level analysis which brings about flexibility in parameter estimation. The hierarchical Bayesian technique is premised on assuming that hierarchical prior distributions are independentlydrawn from the same distribution with unknown parameters. A hierarchical prior for random coefficient (heterogeneous) model is one which assumes that parameters across the units are independent of one another and treated both the mean and the covariance as unknown parameters that require their own prior distributions, James and Stein(1960), Efron and Morris(1975) and Morris(1983).

Therefore, this study derives a hierarchical Bayesian estimator to estimate parameters of dynamic heterogeneous panel modelwhich is an essential task in economics, suited for inference in micro panel data models.

1.2 Nature of Panel Data

The concept of panel data model is a little different from that of cross-sectional and time series data. Each of the cross-sectional and time series has only one dimension, respectively the individual and time. Panel data is possessive of the individual characteristic of the cross-sectional data and time series data. In panel data model, y_{it} denotes the observed values of y for individual i at time t. It is worth mentioning that the word individual connotes the units under study. Such units could be households, countries, firm and so on. Usually, i runs from 1to N and t runs from 1to T implying that N individuals and T time periods are involved. A panel is said to be *balanced* when T is the same for all individuals, else it is said to be unbalanced. An unbalanced panel may result from unavailability of some individuals after some time. If the individuals are humans, this could be as a result of death and if the individuals are firms, it could be due to shut down. However, the balance panel data are employed under this study.

1.3 Micro and Macro Panel Data

Micro panel data refer to the panel data in which the number of units is much greater than the number of time periods (N>T). Typically, many microeconomic analyses use large datasets with many individual to solve economics problems. Micro panels are short and wide because N is much larger than T. Using panel data models of this type can account for unobserved individual-specific differences, or heterogeneity. Micro panels are rich in information, and require the use of considerable quantity of computing power, Olubusoye *et al*.(2016). Working with micro panel datasets invariably present two problems: the problem of unobserved individual- specific effects and the problem of sample selection. These problems may be solved using micro panel data models where each unit of analysis is observed more several times.

Macro panel data refer to panel data with relatively large N and large T. Peter Kennedy describes this situation as “Long and wide” datasets. For example, Penn World Table provides purchasing power parity and national income accounts converted to national prices for 189 counties for some or all the year 1950-2007, which roughly characterize as having both large N and large T, Olubusoye*et al*.(2016). With large *N*, large *T*,macro panels give more attentions to non-stationarity analyses. Specifically, time series fully amendestimation method that accounts for endogeneity of the regressors and correlation among the parameters and heteroscedasticityof the error terms which can now be combined with fixed and random effects panel estimation methods. Some of the prominent results that are obtained with nonstationary macro panel datasets are estimators that have normal limiting distributions and many test of statistics. This is in contrast to the nonstationary time series study where the limiting distributions are complicated functional of Weiner processes. Macro panel data have a longer time series and unlike the problem of nonstandard distributions typical of unit roots tests in time-series analysis.

1.4 Benefits of Panel data

Hsiao (2003) and Baltagi (2005) list several benefits derived from panel data. These include the following:

(i) Controlling for individual heterogeneity: Panel data propose that individuals, firms, states or nations are heterogeneous in nature. On the other hand, time-series and cross-sectional data do not control for heterogeneity hence run the risk of producing biased results. Not controlling for the unobserved individual-specific effects leads to bias in the resulting estimates. Panel data containing time series data for a number of individuals is optimal for investigating the "homogeneity" versus "heterogeneity" consideration.

(ii) Identification of parameters: Panel data are better able to discover and assese effects that are simply not noticeable in pure time series or pure cross-sectional data. Measurement errors can lead to under- identification of an economic model. The availability of multiple data for a given unit or at a given time may allow a researcher to make different transformation to induce different and deducible changes in the estimators.

(iii) Panel data are better able to study the dynamics of adjustment: Panel data are more appropriate for the study of complex issues of dynamic behaviour. Cross-sectional studies that look relatively stable hide a multitude of changes leading to bias in parameter estimation. For example, with cross-section data one can estimate the rate of unemployment at a particular period in time, while panel data can be used to study changes in this proportion over time.

(iv) Analysis of nonstationary time series: When time series data are not stationary, the large sample approximation of the distributions of the maximum likelihood or the least-squares estimator is no longer normally distributed. But if observations among cross-sectional individuals are independent and panel data are accessible, then the central limit theorem across-sectional units can be introduced to show that the limiting distributions of many estimators remain asymptotically normal, Hsiao(1991).

(v) Panel data provide more informative data, more variability and less collinearity among the variables: Panel data sets give more informative data, less collinearity among the variables, more variability, more efficiency and more degrees of freedom. Time-series studies are often plagued with multicollinearity; for example, in the case of demand for cigarettes in United State of America, there is high collinearity between price and income in the aggregate time series data. Therefore, larger sample sizes due to pooling individual and time dimension give accurate statistical inference about the parameters in the model, Baltagi (2005).

1.5 Problems of Panel Data

The problems of panel data in econometrics include *measurement errors*. Panel data models are distorted due to measurement errors. They may arise due to unclear questions, memory errors, and deliberate distortion of responses, misreporting of responses, inappropriate informants, coding inaccuracy and interviewer effects, Kalton *et al*(1989). Also, *short time-series dimension problem*. A typical panel data involve in annual observation (low frequency data) covered a limited span of time for each individual. This means that asymptotic arguments rely heavily on the number of individuals in the panel tending to infinity, Baltagi(1995). Increasing the time span of the panel will increase cost. In fact, this aggravates the chances of attrition and increases the computational problems for limited dependent variable panel data models.

The problem of *non-response* is another problem encountered in the use of panel data. Non-response occurs when one or more questions are left unanswered or are found not to provide a useful response. This may show bias in some surveys, the respondents may refuse to respond faithfully, and the interviewer may not find anybody at home. This may also introduce some bias in the statistical inference drawn from the sample, Baltagi(2004). While non-response can also occur in cross-sectional data, it is more problematic in panel data because subsequent of the data pooling (panel) are still subject to non-response. More so, the problems include attrition and self-selectivity.

*Data collection, management of panel surveys*and *design*are other issues encountered in panel data. This may include interview spacing, incomplete account of the population of interest, time-in-sample bias, reference time, frequency of interview, the use of bounding problems of coverage and respondent not remembering correctly (recall).

Finally, *problems of cross-sectional dependence* are many times found in panel data. Macro panels on firms, countries or regions with long time series that do not give account for cross-country dependence may seriously lead to bias in statistical inference. Nonstationary panels show that several panel unit root tests proposed in the literature assumed cross-section independence. Accounting for cross-section dependence turns out to be essential and improve inference. Panel data is not a solutionor remendy to all the problems that a cross sectional study cannot handle, Olubusoye *et al*. (2016).

1.6 FrequentistMethodversus Bayesian Method

There are many things remotely controversial about statistical analysis due to the mode of estimating parameters. Nevertheless, appearances can be misleading, and a disagreement exists at the very heart of the researcher between the so-called Frequentist (also known as Classical) and Bayesian statisticians.

Frequentist uses methods such as Maximum Likelihood (ML), Ordinary Least Squares (OLS), Instrumental variable (IV), Least SquaresDummy Variable (LSDV), Generalised Least Squares (GLS), Generalised Methods of Moments (GMM) and many others, Silvio (2002). These are the traditional types of method that the research analysts employ for estimation, hypothesis testing, prediction, confidence intervals, and so on. In contrast, Bayesian statistics looks much different, and this is because it is fundamentally all about making partial changes to conditional probabilities; it uses prior distributions for unknown parameters with the appropriate likelihood function of the observed datawhich lead to posterior distributions using the laws of probability, Tsai(2004). Bayesian econometrics is all about the rules of probability.

The classical study considers that the status of parameters is either random or fixed. Fixed effects are treated as parameters and estimated by a least-squares dummy variable, this approach assimilates random effects to the error terms and estimated the effects by a generalised least squares, Wallance and Hussain(1969). While the Bayesian framework treats both fixed and random effects as random variables. Bayesian econometricians specify what is called a “prior distribution” to represent prior parameter information and then update this distribution to be completely consistent with the observed data (using Bayes’ Rule), Egerton (2015).

Subjective uncertainty is an essential component in applied economics and economic theory, it characterizes the impressions of economic researchers about the status of their environment. Subjective uncertainty in applied economics explains the situation of researchers who judge competing models based on the conditions of decision makers who must act given short information and their implications for what might be observed, Poirier (1995). With the uses of the expected utility model in increasingly richer environments, explicit distributional assumptions have become common, but closed form analytical expressions for the distribution of observables are typically unobtainable. In this situation, simulation methods are required which represent probability distributions by related finite samples. Even in the commonest typical situation the decision maker must precede knowing the observe values which are random variables in models, but not knowing the specification of technology. Bayesian inference reviews the applied economics problem in this way: given a probability distribution over the study models and the prediction of each model for the observe values, the distribution of the models *conditional* on the observed values is then defined.

A fundamental issue in any form of inference, either classical or Bayesian is explicitly based on probability theory of some kind. The set of models which investigators, researchers and theorists have before them is constantly changing and needs attention.

1.7 The Concept of Bayesian Inference

Berka *et al* (2011) examined a conventional set up of a Bayesian statistics as y; a random variableknown to have a probability distribution function conditional on an unknown β parameter, that is, $P(y \mid \beta)$.The focus is on β parameter value, given one or more independent draws from the conditional distribution of y given β . Also, prior beliefs about the value of the β parameter are captured in a prior probability distribution then combining the sample information and this prior distribution using Bayes' theorem to obtain posterior distribution, which is the conditional distribution of the parameter given the data $\left[P(\beta \mid y)\right]$

In addition, Bayesian analysis is of two settings. Firstly, data in the model given some unknown constant parameters indicated as a probability distribution function: $p(\beta, y)$, this function is referred to as the likelihood function denoted by $L(\beta)$ or $L(\beta / y)$. Secondly, parameters in the model have a specific prior distribution denoted as $p(\beta)$. The researchers choose prior distribution carefully to maximise its impact on posterior distribution. Then, the posterior distribution which is the conditional distribution of the parameter given the datais calculated using Bayes' theorem.

$$p(\beta / y) = \frac{p(\beta, y)}{p(y)} = \frac{p(y \mid \beta) p(\beta)}{p(y)} = \frac{p(y / \beta).p(\beta)}{\int p(y / \beta).p(\beta) d\beta} \tag{1.6}$$

The quantity $p(y)$ is called marginal distribution normalizing constant of the posterior distribution, and as long as it is finite it can be ignored. Hence $p(\beta \mid y)$ is written as:

$$p(\beta \mid y) \propto p(y \mid \beta) p(\beta) = L(\beta).p(\beta). \tag{1.7}$$

Once the product in equation (1.7) is calculated, then, the resulting expression will be integrated into one as a function of the constant parameter. At this stage itrequires little effort to figure out and realised the class of the posterior distribution otherwise a simulation technique is introduced. Bayesian models are concerned with the inferences on a parameter set $\beta = (\beta_1, \beta_2, ..., \beta_q)$, of dimension q that includes uncertain quantity, whether fixed or random effects, hierarchical parameters, unobserved indicator variables and missing data, Gelman and Rubin (1992).

In econometrics, there are typically given reasons for lack of Bayesian approaches. First is the problem in choosing correct prior distributions, another is theanalytical complexity of deriving posterior distributions as well as the need for a specified parametric model. None of these aforementioned reasons is compelling. In principle, the influence of specification of the prior distribution vanishes as the sample gets larger, as Imbens and Wooldridge(2002)formalized the Bernstein-Von Mises theorem. This is able to be compared to the way in which large sample normal approximations can be applied for the finite sample distributions of frequentist methods. On the other hand, it is likely that the sampling distribution of the maximum likelihood estimator is not well approximated by a normal distribution against the true value of the parameter in a classical analysis, if the posterior distribution is sensitive to the choice of prior distribution. In Bayesian analysis it is requiredthat a prior distribution on all the parameters of the model is fully specified. In classical analysis, it is possible to specify only part of the model and use a semi-parametric approach. This benefit is not as clear cut as it may seem. Koop and Poirier(2004) suggests when the greatest questions under consideration do not depend on certain characteristics of the distribution, the outcomes of a parametric model are often strong given an inconstant specification of the inconvenience functions. A semi-parametric model as a result,extends to a complete parametric model with flexibility in modelling the nonparametric model often works well in empirical analyses, Yatchew(1998). Bayesian analyses are now in practice in many settings therefore, few restrictions on the dimension of the models used and type of prior distributions can now be considered. In particular, Bayesian approaches are attractive in studies with many parameters for example discrete choice observation with many unobserved product features, instrumental variables with many instruments and panel data with individual-level heterogeneity with many parameters. In such cases, there will be poor repeated sampling properties in the methods that estimate every parameter accurately without connecting it to identical parameters. This reflects the dogmatic posterior distributions obtaining from flat prior distribution (non-informative prior)in Bayesian analyses.Imbens and Wooldridge(2007) proposed a more attractive method that is suitable to the aforementioned examples based on hierarchical prior distributions where the parameters in the model are drawn from a common distribution with unknown location and scale independently.

Under hierarchical Bayesian reasoning, whenever belief about the behaviour of the situation and the appropriate prior information are available and known to the researcher, such information may be desirable to make use of in the estimation of the regression parameter model. This is extra structure if consistent with the patterns in the data making the estimates to be more precise and accurate, McCulloch and Tsay(1994), Rossi *et al*.(1996) and Hansen *et al*.(2006).

1.8 Specifications of Prior Distributions

Prior distributions are the key components in a Bayesian model analysis and should be chosen carefully. Prior information are meant to show and quantify any information the researcher has before seeing the data which he wishes to inculcate in the data analysis. Hence, the prior distribution is a mechanism that enables the researchers to incorporate expert knowledge into the analysis and to combine that information with the available observed data. Priors can take any reasonable form of distribution. However, it is usual to choose specific classes of prior distribution that are easy to interpret when multiplied with the likelihood which would make computation easier, Gelman (2006). Conjugateand natural conjugate prior distributions typically have such advantages.

Raif and Schlaifer(1961) defined a*conjugate prior distribution* as one which, when multiplied with the likelihood function gives a posterior distribution that has the same class with prior distributions. A natural conjugate prior has the additional property of having the common functional form as the likelihood function of the data. This property means that the prior information can be interpreted in the same way as the likelihood function's information of the study. In other words, the prior can be understood as arising from an imaginary dataset from the same process that produced the actual data.

Some statisticians want to benefit from the Bayesian approach with as short as influence from the prior distribution as possible which can be obtained by choosing priors that have a limited influence on the posterior distribution. Such priors are referred to as noninformative priors, and they are common to some study. If the posterior distribution is flat a prior distribution is noninformative. The use of noninformative prior can lead to improper posterior which is nonintegrable and possibly not making good inferences. Improper prior distributions are commonly used in Bayesian methods for they give noninformative prior and proper posterior distribution, Rao(1982). Noninformative prior does not represent complete ignorance about the parameter under study. A noninformative prior distribution projects a very high variance and does not impose strong preconditions on the parameter in question and as a result data dominate the posterior distribution completely. An informative prior can be used if nothing or less information is known of the parameter model before the experiment, Kalton *et al* (1989). However, informative priors refer to information we have about the parameters present in the model before seeing the data. An informative prior dominates the likelihood function, and thus it has a credible influence on the posterior distribution.

Nevertheless, if the posterior distribution for the parameter vector has no analytical closed form, a hierarchical Bayesian analysis can be applied using a sampling-based approach, such as the Gibbs sampler, Metropolis-Hastings algorithm, Importance Sampling, and so on.

1.9 The Likelihood Function

The likelihood function is the density of the data conditional on the parameters of the model, $p(y\,|\,\beta)$. Koop(2003) referred to the likelihood function as the data generating process. Hence, it consists all the information about the parameters of the model.

Let $X^n = (X_1, X_2, ..., X_n)$ having joint density $p(X^n;\beta) = p(X_1, X_2, ..., X_n;\beta)$ where $\beta \in \Theta$.

The likelihood function $L: \beta \to (0, \infty)$ is defined by $L(\beta; x^n) = p(x^n, \beta)$ where x^n is fixed and β varies in Θ. The likelihood function is a function of β and it is not a probability density function. If the data are independent and are identically distributed, then the likelihood is given as:

$$L(\beta) = \prod_{i=1}^{n} p(x_i; \beta) \tag{1.8}$$

A constant of proportionality is defined only up to likelihood function. It is used to generate estimators like the maximum likelihood estimators and it is a pivotal clue for Bayesian inference, Daniel (1997).

1.10 The Posterior Distribution

The posterior distribution is a probability model which describes the knowledge obtained after observing a set of data, $p(\beta | y)$. Inferences about the unknown parameters are derived from the posterior distribution when Bayesian analysis is conducted. It is very common in Bayesian inference that the posterior distribution of the parameters is analytically intractable, Tanner and Wong(1987). This means that it is not possible to derive a closed form summaries of the posterior, such as mean, variance, or marginal distribution of a specific parameter. Most often, the posterior density is only known up to a normalizing factor. This problem will be overcome if the standard practice is resort to simulation methods. For instance, if a researcher draws a random sample of independent and identically distributed sample from the posterior distribution of parameters, then, using the standard Monte Carlo method, the means of function having finite posterior expectation can be approximated numerically by a simple average, West and Harrison (1997). To simulate a Markov chain in order to obtain posterior samples from the joint posterior distribution of parameters under a certain prior probability density, Metropolis *et al.* (1953) and Hasting (1970) developed the theory of Markov Chain Monte Carlo (MCMC). Precision of the prior and the likelihood are the relative weights which lead to the posterior distribution of β. Hence, the posterior distribution is written as:

$$p(\beta | y) \propto p(y | \beta) p(\beta) = L(\beta).p(\beta). \tag{1.9}$$

1.11 Justification for the Study

In recent times, many studies have been conducted in the estimation of static panel data model using different classical estimation techniques such as Ordinary Least Squares (OLS), Maximum Likelihood (ML), Instrumental Variables (IV) and many others. They appear to be the most favourable forms of consistent estimation method for both fixed and random effect specifications which are found to be inconsistent for large N and small T. In the estimation of static panel data model, error terms are assumed to be homoscedastic through the random individual effect, a condition which is often violated in most economic models. Therefore, the dynamic panel data model is specified which presumes there is correlation between error terms and lagged dependent variable, characterising the heterogeneity among the units. The use of classical approach to estimate parameters of dynamic panel data model is however usually plagued with the problem of not controlling for the heterogeneity leading to inconsistent estimates of the parameters. The performance of several estimators for dynamic panel data models in the context of macroeconomics has been analysed by different authors, namely: Baltagi and Levin (1986), Ahn and Schmidt (1995), Judson and Owen (1996), Senykina and Wooldridge (2011) which failed to address the problem of heterogeneity of the parameter error variance across individuals.

Obtaining consistent estimates, especially in the presence of heterogeneity among the regressors, is a severe problem in dynamic panel data models which has been shown by Pesaran and Smith (1995) that not accounting for the heterogeneity produces inconsistent estimates of the mean autoregressive coefficient and pooling of observations will result in asymptotic non-consistent estimates.

Therefore, this study attempts to account for heterogeneity of the parameter error variance across the individuals of dimensions of N and T ($N<T$, $N=T$ and $N>T$) in which no single study has been able to compare the posterior estimates through the derived hierarchical Bayesian estimator.

Furthermore, there are inadequacies in the use of distribution property such as normal and logitnormal distribution whose support is not imposing a stability condition on the coefficient of lagged dependent variable.To impose stationarity on autoregressive order one coefficient (δ_i), we assume that the δ_i are generated from a distribution whose support is (0, 1), in particular the beta distribution.

The hierarchical Bayesian approach allows the study to free up normality assumptions which brings about flexibility in parameter estimation of a high dimensional space. This hierarchical Bayesian technique based on hierarchical prior distributions for random coefficient (heterogeneous) model is one which assumes that parameters across the units are independent of one another and treated both the mean and the covariance as unknown parameters that require their own prior distributions.

1.12 Aim and Objectives of the Study

The aim of this study is to derive a hierarchical Bayesian estimator of dynamic panel data model with heterogeneity among the units as an extension of the work of Koop (2003) and Zhang and Small (2006).

The specific objectives are:

i. To examine the performance of hierarchical Bayesian estimator as the dimensions of N and T change.

ii. To employ stability condition on coefficient of lagged dependent variable using beta distribution (0, 1) in examining its performance on the posterior estimation.

iii. To investigate the prior sensitivity of the parameters to posterior probability of the model using relatively non-informative and informative priors.

1.13 Scope of the Study

This study focuses on hierarchical Bayesian estimation of dynamic panel data model in the presence of heterogeneity among the units. The hierarchical Bayesian estimation is derived in two stages: the first stage of hierarchical parameter priors are $\gamma_i \mid y,\ h \sim N(\mu_\gamma,\ V_\gamma)$ and $h \sim G(\underline{s}^{-2}, \underline{v})$ with independent Normal-Gamma prior which are obtained by the second stage of hierarchical parameter priors $\mu_\gamma \sim \boldsymbol{N}(\underline{\mu}_\gamma, \underline{\Sigma}_\gamma)$ and $V_\gamma \sim \boldsymbol{W}\left(\underline{\boldsymbol{v}}_\gamma,\ \underline{V}_\gamma^{-1}\right)$ with independent Normal-Wishart prior to produce consistent estimates that account for heterogeneity. Also, the autoregressive order (1) coefficient for each unit of lagged dependent variable is assumed to have absolute values less than 1 that is $|\delta_i| < 1$, generated from a beta distribution whose support is (0,1) to facilitate the Bayesian sampling properties. Relatively non-informative prior and informative prior are used to examine the sensitivity of prior information on the posterior estimates. The posterior inference for the study model (dynamic panel data model) is carried out using Markov Chain Monte Carlo (MCMC) approach through Gibbs sampler algorithm which involves data generating process.

1.14 Organisation of the Thesis

The thesis consists of five chapters. Following this chapter is the chapter which reviews literature on statics panel data, dynamic panel data,random coefficient (heterogeneity) panel data and hierarchical Bayesian estimation.Chapter three contains theoretical framework: Linear regression panel data model, Set-up of Bayesian estimation of panel data, hierarchical Bayesian computation. Analysis of data and interpretations are discussed in chapter four while chapter five presents the summary of findings, conclusions and recommendations along with research contribution to knowledge, recommendations and suggestions for further research.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews related literature on static panel data, dynamic panel data,random coefficient specification (heterogeneity) panel data and hierarchical Bayesian estimation.

2.2 Static Panel Data

Static panel data models are traditional models which assume that the error terms have constant variance (homoscedastic) and non serial correlation via the random individual effects, Hsiao (2003a) and Baltagi (2002). In the literature, serial correlation of the linear panel data models biases the standard errors and causes the results to be less efficient. A number of tests for serial correlation in panel data models have been proposed by Wooldridge (2002) which are very attractive, easy to implement and require relatively few assumptions. Tests for serial correlation in the presence of fixed and random effects are extensively discussed by Baltagi (2001). The problems of serial correlation in panel data are well noted among Lillard and Willis (1978), Bhargava *et al*. (1982) and Baltagi and Li (1991, 1994, and 1995).It is evident from most studies that problems in the panel data analysis are mainly on serial correlation and heteroscedasticity. Meanwhile, when the attention is on the serial correlation, heteroscedasticity is ignored, and when the focus is on the heteroscedasticity, serial correlation is ignored. It is very rare in literatures to consider both serial correlation and heteroscedasticity problems in the analysis of panel data but Baltagi (2008) in his work assumed the existence of both serial correlation and heteroscedasticity problems in panel data regression model. He derived a joint Lagrangian Multilpier (LM) test for homoscedasticity and zero order serial correlation. In the context of random effects panel data model, he developed a conditional LM test for homoscedasticity, given serial correlation and a conditional LM test when there is zero order serial correlation; given homoscedasticity. The results via Monte Carlo simulation showed that the tests along with their likelihood ratio alternatives have good size and power under various forms of heteroscedasticity.

Garba *et al*. (2013) investigated the efficiency of methods of estimating panel data models when the assumptions of homoscedasticity, no collinearity and no autocorrelation are violated. Also, Olofin *et al*. (2010) considered time effects to test for the feasible existence of serial correlation and heteroscedasticity in the model and a more comprehensive model, the two-way error component model that takes care of both random individual.

The general framework for the static panel data model is given as

$$y_{it} = X_{it}\beta + \varepsilon_{it} \tag{2.1}$$

and

$$\varepsilon_{it} = \varepsilon_i + \eta_{it} \tag{2.2}$$

Where $i = 1,2,...,N$ and $t = 1,2,...,T$ for N and T are the cross-section and time series dimensions respectively, also y_{it} is the response variable, the slope coefficient (β), X_{it} is the independent variable, and ε_{it} is error term.

Consider one way error component model as shown in (2.2),the error term ε_{it} is decomposed into ε_i and η_{it}, where ε_i is the individual-specific effect (that captures the individual heterogeneity) and η_{it} is the error term which varies over the cross-section and time. The assumptions that guide the error term in the static panel data model are: $E(\varepsilon_i) = 0$, $E(\eta_{it}) = 0$, $E(\varepsilon_i\eta_{it}) = 0$, $E(\varepsilon_i^2) = \sigma_\varepsilon^2$, $E(\eta_{it}^2) = \sigma_\eta^2$, and $E(\eta_{it}\eta_{is}) = 0$, $t \neq s$.

Also, for the independent variable to be exogeneous under the assumptions of the error terms, $E(\varepsilon_i X_{it}) = 0$ and $E(\eta_{it}X_{is}) = 0$, for $t \neq s$ and $t = s$.

To obtain a consistent estimator of β, Ordinary Least Squares (OLS) method can be used.

2.3 Dynamic Panel Data

A dynamic panel data model can be described as model with lagged dependent variable on the right-hand side of the equation of a panel data model**.** Dynamic panel data are exhibiting phenomenal growth and consistently becoming popular among the behavioural and social researchers.

The linear dynamic panel data model which contains the lagged dependent variable $y_{i,t-1}$ and the independent variables x_{it} is of the form:

$$y_{it} = \delta y_{i,t-1} + X'_{it}\beta + \eta_i + \varepsilon_{it} \qquad (2.3)$$

Where $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$ for N and T are the cross-sectional and time series dimensions respectively. Also, y_{it} is the dependent variable, β is unknown parameter vector of the *k* explanatory variables, X'_{it} is row vector of explanatory variables with dimension *k*, η_i is individual specific fixed effects, δ is unknown parameter of the lagged endogenous variable, $|\delta| < 1$ and $\varepsilon_{it} \sim N(0, \sigma^2_\varepsilon)$,Islam (1995). The model in (2.3) is specified based on two assumptions: the error term is uncorrelated with the lagged dependent variable that is $E(y_{i,t-1}\ \varepsilon_{it}) = 0$, and the error term is orthogonal to the exogenous variables, that is $E(X'_{it}\ \varepsilon_{it}) = 0$

Dynamic panel model has been commonly used recently in empirical economic studies. Examples are Lee *et al*. (1997) that studied the convergence of countries economic outputs and Pesaran *et al*. (2004) which examined the determinants of economic growth using panel of countries. Bhargava (1991), who studied the income elasticity of the demand for food and nutrients in rural South India using panel of households. Baltagi and Levin (1986) studied the effect of taxation and advertisement on cigarette demand using a panel datasets in the United State of America,

The discussion of Dynamic Panel Data (DPD) model is opened and suggested to estimate the model with unobserved individual component using Generalised Least Squares (GLS) estimator. However, GLS random effects or Maximum Likelihood (ML) estimators are not consistent if the unobserved individual effects are correlated with exogenous variable, Balestra and Nerlove (1966).

Under asymptotic sequences where the number of cross-sectional units (N) is large and the number of time period (T) is fixed, the within groups estimator of dynamic panel data is biased even in large samples, Nickel (1981). In the past studies, OLS estimator is biased for an analytical treatment, Nerlove(1967). Nickel (1981) and Sevestre and Trognon (1985) derived analytical expressions for the asymptotic biases of the OLS estimator of an autoregressive panel data models with fixed time dimension. Nerlove (1967 and 1971)explored the properties of the bias of the OLS estimation through Monte Carlo simulations. In the random effects model, Random Effect Maximum Likelihood (REML) in levels and first differences estimators enforce mean stationarity for Autoregressive of order p (AR (p)) models with individual effects, their investigation showed that REML in levels achieved substantial efficiency relative to estimators from data in differences. The random effects GLS estimator is also biased in a dynamic panel data model. Therefore, Everaert and Pozzi (2007) considered using Monte Carlo simulations for panels with small to moderate T in the bias correction for the within estimator based on an iterative bootstrap procedure. Hence, the procedure offers a better substitute for existing dynamic panel data. Olajide *et al*. (2014) investigated the performance of GMM and IV methods of dynamic panel data model with random individual effect when the disturbance term is serially correlated. They discovered that Anderson- Hsiao (AH) using lagged difference as instrument is appropriate when the dimension of time is small while Arellano-Bond Generalised Method of Moment (ABGMM) performed better when T is large and the effect of the serial correlation is minimal.

In the fixed effects specification, the estimation of dynamic panel data models has been the major tasks in econometrics. Several literatures have proposed and compared various Instrumental Variables (IV) and Generalised Method of Moments (GMM) estimators of fixed effect dynamic panel data models. Prominent among these are: Anderson and Hsiao (1981, 1982), Amemiya and MaCurdy(1986), Arrelano and Bond (1991), Ahn and Schmidt (1995), Blundell and Bond (1998), Harris and Matyas (2010). Also, Nickell (1981) demonstrates that the Least Squares Dummy Variable (LSDV) for autoregressive panel data models is inconsistent for small T. Woodridge (2005) suggested an approach for handling initial conditions problem in dynamic nonlinear unobserved effect models. The approach has been applied to the probit, Tobit and Poisson panel data models. A modified maximum likelihood to correct the first term on the asymptotic bias that is associated with estimation of fixed effects which reduce the order of bias is proposed by Carro (2007). Although, this estimator is consistent only as T tends to infinity, it is shown through Monte Carlo experiments to have finite sample bias for logit and probit dynamic panel data models with T= 8. Fraderisken *et al*. (2007) suggested estimation of dynamic discrete choice models by using the duration in the current states as a covariate. The estimators (maximum likelihood estimator) allow for group- specific effects in both parametric and semi-parametric versions of the model. The method is applied to analyse job duration allowing for firm-specific effect using Danish data on all employees of all establishments in the private sector observed over the period 1980-2000. Harris and Matyas (2010) provided a survey of the mainstream estimators, in their simulation experiment, they chose the values of N and T to be small. The results showed that within estimator in the fixed effect setting asymptotically performed better than IV proposed by Arellano (1988) and Arellano and Bond (1991).

Moreover, additional difficulties arise when the regression equation includes dynamic terms are connected to the degree of heterogeneity in the lagged dependent variable coefficients.

Under the classical assumptions,a number of estimators have been proposed for estimating a heterogeneous dynamic panel data model. In the pooled cross-sectional and time series data (panel data) models, the Pooled Least Squares (POLS) estimator appears to be the Best Linear Unbiased Estimator (BLUE) in the general linear regression model but inconsistent. Also, Ordinary Least Squares Fixed Effects (OLS-FE) estimator is used to account for some degree of heterogeneity in dynamic panel data models, when estimating individual-fixed effects or random effects when some concomitant assumptions are imposed over heterogeneous “intercepts" and homogeneous “slope coefficients" across individuals. The OLS-FE is asymptotically biased and inconsistent when the heterogeneous slope coefficients are correlated with the variance of the regressors. Literature in econometrics has developed two main approaches to deal with OLS-FE estimator bias.

Firstly, using tools for the lagged dependent variable, and compare four estimators: the First-Difference Generalised Methods of Moments estimator (FD-GMM), Arellano and Bond(1991), the System Generalised Moment of Method estimator (SYS-GMM), Blundell and Bond(1998), the Long-Difference GMM estimator (LD-GMM or LDP-

GMM), the just-identified Instrumental Variable estimator (AH-IV), and Anderson and Hsiao(1981), each of these tools depends on long-difference parameters used by Hahn et al. (2007) and Huang and Ritter (2009).

Secondly, the approach comprising of three estimators such as: a simulation-based indirect inference method (II), Gouriéroux *et al*. (1993), Least Squares Dummy Variable estimator (LSDV), Kiviet (1995), Bruno(2005), and an iterative Bootstrap-based Correction procedure (BC) Everaert and Pozzi(2007) to correct for the estimation bias either by simulation or analytical estimate. Everaert and Pozzi (2007) developed bias correction formulas and advance in theory to reduce the POLS and FE bias. Some problems could arise and render proposed instruments invalid such as non-zero correlation between the regressors and the individual fixed effects leading to unobserved heterogeneity and endogeneity, and presence of residual autocorrelation, which breaks one of the most essential assumptions of the GMM/IV estimators, Arellano and Bond(1991).

As mentioned earlier, especially when the time period (T) is small, the fixed effect estimator is not consistent, Nickell(1981). In such cases, IV estimator proposed by Anderson and Hsiao(1981) and GMM estimator suggested by Arellano and Bond(1991) are both used. These estimators are basically methods that choose parameter estimates such that the theoretical model is satisfied. The estimates are chosen to reduce the weighted distance between the actual and theoretical values. Therefore, it requires that the theoretical relations between the parameters satisfy orthogonality conditions; meaning correlations between the independent variables and instruments is zero.

Baltagi (2005) suggested Limited Information Maximum Likelihood (LIML) as a possible alternative to GMM in order to eliminate the small sample bias of GMM. However, in dynamic panel data models where the cross-sectional data are highly autoregressive and the time-series observation is moderately small, the GMM estimator is found to have poor precision in simulation studies and large finite sample bias.Islam (1998) investigates the small sample size of dynamic panel data estimators using Summers-Hesston dataset for estimation of the growth convergence equations. His Monte-Carlo results proved that parameter estimates from different estimators are not the same.

Bun and Caress (2006) developed bias-corrected estimation method for theLSDV estimator in the dynamic panel data model with heteroscedastic error terms. The inconsistencies of the LSDV estimator for N large and finite T were derived under panel data heteroscedasticity. The results are used to express and extend the existing nonlinear bias correction and additive procedures. In addition, provides some simulation techniques which allow for either cross-section or time-series heteroscedasticity. The results showed that root mean squared criterion bias-corrected LSDV estimators performed well against GMM estimators.

Hayakawa (2008) considered the following panel auto regression (p) model

$$y_{it} = \alpha_1 y_{i,t-1} + \alpha_2 y_{i,t-2} + \ldots + \alpha_p y_{i,t-p} + \eta_i + v_{it} \qquad (2.4)$$

Where y_{it} is the endogenous variable, $y_{i,t-p}$ is the P x 1 vector of lagged endogenous variables, $\alpha_{(1,2,\ldots,p)}$ is unknown parameter of the lagged endogenous variables, and v_{it} is the error term, η is an unobserved individual-specific time-invariant effect in which heterogeneity in the average of the y_{it} series across units is exermined. Where these two assumptions $\eta_i \sim \text{IIN}\,(0, \sigma_\eta^2)$ and $v_{it} \sim \text{IIN}\,(0, \sigma_v^2)$ are independent of each other and among themselves.

When both N and T are large, Hayakawa showed that the infeasible optimal IV estimator using instrumented deviated from past means are asymptotically the same in the sense that IV and GMMestimators have the identical asymptotic distribution. He further revealed that if normality assumption is on the error terms, the suggested IV estimator is asymptotically efficient when both N and T are large. The simulation results indicated that in terms of the bias and median absolute error, the new IV estimator performs better than the GMM which is commonly used in the literature.

Garba *et al*. (2013) performed Monte Carlo experiments based on panel data model. They compared the First-Differenced (FD), Between Estimator (BTW), Feasible Generalised Least Squares (FGLS) and Pooled Ordinary Least Squares (POLS) estimators. Their findings for several combinations of violations revealed that in small sample sizes, irrespective of number of time spans, FGLS is preferable when heteroscedasticity is intense regardless of level of autocorrelation. But when heteroscedasticity is low and the autocorrelation level is moderate, both FGLS and FD are preferred, while BTW performs better only when there is low degree of heteroscedasticity and no autocorrelation. That is, in large sample sizes with little time spans both FD and BTW could be used when there is no autocorrelation and low degree of heteroscedasticity. Also when the degree of heteroscedasticity is not severe and there is no autocorrelation in large sample sizes with small time periods, either the FD or FGLS would produce efficient results. Finally, FGLS is superior when severe degree of heteroscedasticity is present and autocorrelation is visible in large sample observation regardless of time periods. Meanwhile, both FD and FGLS are appropriate when there is low heteroscedasticity despite the existence of autocorrelation and multicollinearity.

However, Blundell and Bond, (1998) noted that the aforementioned estimators bothencounter a weak instrument problem when the nature of the estimator depends on small T and the dynamic panel autoregressive coefficient tends to unity, invariably the estimators are asymptotically random, and when Tis large the unweighted GMM estimators are inconsistent and the two-stage least squares estimator may violate economics assumptions. Franzese and Hays (2007) compared both the performance of the IVestimator and ML estimator of the panel data models with a spatially lagged dependent variable in terms of unbiasedness and efficiency but without considering spatial fixed/random effects. The results showed that the ML estimator is unbiased and inefficient.

Wansbeek and Knaap(1999) considered a dynamic panel data model with heterogeneous coefficients on the lagged dependent variable and the time period, given as:

$$y_{it} = \delta y_{i,t-1} + \eta_i + u_{it} \tag{2.5}$$

Where y_{it} is an observation on series for individual (unit) i in time t, (dependent variable), $y_{i,t-1}$ is the one-period lagged value of the dependent variable with parameter δ. η is an unobserved individual-specific time-invariant effect in which heterogeneity in the average of the y_{it} series across units is exermined, and u_{it} is the error term. The two assumptions $\eta_i \sim \text{IIN}(0, \sigma_\eta^2)$ and $u_{it} \sim \text{IIN}(0, \sigma_u^2)$ are independent of each other.

The above model results from Islam's (1995) version of Solow's model on growth convergence among countries. Wansbeek and Knaap (1999) showed that double differencing solved the problems of the individual specific effects (η_i) on the first

stage of differencing and heterogeneous coefficient on the time period (ε_i) on the second stage of differencing. The most prominent assumption for panel data models is that the individuals in the database are generated from a population with the same regression coefficient vector. Obviously, classical econometric approach appeared to be unbiased, inconsistent and highly inefficient in literature.
Therefore, Bayesian linear regression is an alternative method to classical linear regression in which the statistical analysis is undertaken within the context of Bayesian inference.

2.4 Random Coefficient (Heterogeneity) Panel Data

Panel datasets derive their effectiveness through estimating and identifying effects that are not noticeable with data that are cross-sectional or time series. Particularly, pooling of data realises a deep analysis and yields a good source of variation that permits effective parameter estimation. Hence, more reliable estimates could be obtained and sophisticated behavioural models with less restrictive assumptions can be tested.

According to Alcacer *etal*.(2013), the assumption for panel datasets is that the units in the database are usually generated through a population with an identical regression coefficient vector. Conversely, the slope coefficients of a panel data model must be fixed and cogent. This assumption is not supported by most economic models. However, when such assumption is relaxed, the panel data models are studied and the model is referred to as “Random-Coefficients Panel Data (RCPD) model". Swamy (1970) is one of the earliest authors to introduce variability in the coefficients in a panel data setting and parameterise them assuming independence between the random coefficients and the regressors and a random coefficient framework. Swamy, in his different publications, examined RCPD model. A number of econometrics and statistical publications made reference to this model as the Random Coefficient Regression (RCR) model or Swamy’s model. He assumes that in RCR model, the individuals in the panel datasets are generated via a population with an identical regression coefficientthat has a fixed and a random component which enable slope coefficients to vary from individual to individual.

There are twocategories of random-coefficients models, namely: stationary random-coefficients and non-stationary random-coefficients models. These models depend on the assumption about the coefficient variation. Stationary random-coefficients models are the regression coefficients having constant means and variance-covariances. While non-stationary random-coefficients models are the regression coefficients having non constant means and variance which can vary systematically. These two models are important and appropriate for modeling the systematic structural variation in time series data, for example the Cooley-Prescott (1973) model.

The random-coefficients models, in general, have been used in diverse fields of econometric and established a unifying setup for many statistical problems. Several literatures have proposed the use of RCR model, among many are Swamy's models which severally have been used in finance and economics.

Pesaran and Smith, (1995) suggested another estimator that improved Swamy's estimator called a Mean Group (MG) estimator which is computationally and analytically simple. A mean group estimator for estimation of dynamic random coefficient panel data models is defined as the simple average of the parameters of the model:

$$y_{it} = \delta_i y_{i,t-1} + \beta_i X_{it} + \varepsilon_{it} \, , \; i = 1,2,...,N \, , \; t = 1,2,...,T \qquad (2.6)$$

Where y_{it} is the endogenous variable , X_{it} is a k× 1 vector of exogenous variables, β_i is the slope coefficient, viewed as invariant over time, but varying from one unit to another, $y_{i,t-1}$ is the lagged endogenous variable, δ is unknown parameter of the lagged endogenous variable, and the disturbance term ε_{it} is assumed to be independently, identically distributed over t with mean zero and variance σ_i^2, and is independent across i.

Let $\Theta_i = (\delta_i \beta_i)'$. We assume that Θ_i is independently distributed across i with

$$E(\bar{\Theta}_i) = \bar{\Theta} = (\bar{\delta} \;\; \bar{\beta})' \qquad (2.7)$$

$$\Omega = E[(\Theta_i - \bar{\Theta}_i)(\Theta_i - \bar{\Theta}_i)'] \qquad (2.8)$$

MG estimator is described as the simple average of the OLS estimator, $\hat{\Theta}_i$:

$$\hat{\bar{\Theta}}_{MG} = \frac{1}{N} \sum_{i=1}^{N} \hat{\Theta}_i \qquad (2.9)$$

When the error terms are independently distributed and regressors are strictly exogeneous, an unbiased estimator of the covariance matrix of $\hat{\bar{\Theta}}_{MG}$ can be estimated as:

$$\widehat{Cov}\left(\hat{\bar{\Theta}}_{MG}\right) = N^{-1}\hat{\Omega}^{*} \tag{2.10}$$

Where

$$\hat{\Omega} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\Theta}_i - \frac{1}{N}\sum_{j=1}^{N}\hat{\Theta}_j\right) \tag{2.11}$$

$$= \left(\hat{\Theta}_i - \frac{1}{N}\sum_{j=1}^{N}\hat{\Theta}_j\right)' - N^{-1}\sum_{i=1}^{N}\hat{\sigma}^2 (X_i' X_i)^{-1} \tag{2.12}$$

$$\hat{\Omega}^{*} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\hat{\Theta}_i - \frac{1}{N}\sum_{j=1}^{N}\hat{\Theta}_j\right)\left(\left(\hat{\Theta}_i - \frac{1}{N}\sum_{j=1}^{N}\hat{\Theta}_j\right)'\right) \tag{2.13}$$

The mean group estimator is consistent when both N and $T \to \infty$. In finite T, $\hat{\Theta}_i$ for Θ_i is biased to the order of T^{-1} Hurwicz (1950), Kiviet and Phillips, (1993). The limited Monte Carlo study appears to show that the MG estimatorcan be critically biased when T is very small, Hsiao *et al.*(1999).

The Swamy's GLS estimator is another estimator equivalent to MG estimator as T tends to infinitywhen the model is linear in the variable slope coefficients. It is noted that when random coefficients are assumed, the subject parameters problem is no longer applicable but T small sample bias in question remains valid in the heterogeneous panel data model. Pesaran and Yamagata (2008) demonstrated a test for slope coefficient homogeneity in macro panel data. Arellano and Bonhomme (2011a) managed the general identification of random coefficients in a fixed effects approach when the distribution of the variable slope coefficients is not defined and when possible endogeneity of regressors with respect to the individual-specific coefficients is allowed in the model that is linear in coefficients. Bonhomme(2012) considered a random coefficient model to be linear in parameters and illustrated the functional differencing approach for the case.
Mohamed (2016)provided a generalised model for the random-coefficients panel data model where the error terms are cross-sectional heteroscedastic and correlated with the

first-order autocorrelation of the time series errors. Heemphasisedthatthe usual estimators such as Generalised Least Squares (GLS), Mean Group (MG) and General Mean Group (GMG) estimators are not suitable for the standard random-coefficients panel data model. Therefore, he considered the MG and GMG estimators to be less efficient to RCR estimator in random- and fixed-coefficients models especially when T is small.

Lin and Ng (2011) proposed two techniques for estimating panel data models with unit specific parameters when unit membership is unknown. The first approach applies the time series framework of the individual slope coefficients to generate threshold variables. The second approach is a modification of the K-means algorithm in which the units are grouped according to their differences. Both approaches are uncertain about the origins of parameter heterogeneity. Hsiao and Tahmiscioglu (1997) discovered heterogeneity in the parameters of the investment dynamics and observed that differences in parameter cannot be explained by commonly considered firm characteristics. Browning and Carro (2007) pointed out that researcher allow less parameter heterogeneity than those existing in empirical study. Robertson and Symons (1992) found that using the estimator proposed by Anderson and Hsiao (1982) was severely biased when parameter heterogeneity is excluded. Through a random coefficient model, the mean of the coefficients can be estimated but is uninformative about the response at a more disaggregated level, which many times are object of interest.

Holtz-Eakin *et al*.(1988) and Ahn *et al*.(2001) focused on single factor residual models and allowed for time-varying individual effects in the case of panel model with homogeneous slopes where $N \to \infty$and T is fixed. Robertson and Symons (2002) studied a random coefficient multi-factor error model where the factors are distributed independently of explanatory variable, x_{it} , and argued that the MLestimator would still be appropriate even when N is greater than T.

Pesaran (2006) proposed the Common Correlated Effects (CCE) approach of multi-factor error structure to estimate panel data models which were developed by Chudik*et al.* (2011), Pesaran and Tosetti (2011), and Kapetanios *et al*. (2011). The CCE method is efficient to possible unit roots in factors, different types of cross-section dependence of errors, and slope heterogeneity. However, Pesaran (2006) extended the CCE approach to allow for the dynamic panel data model or weakly exogenous variables as regressors. While Chudik and Pesaran (2013) extended this approach but not straightforward for the parameter heterogeneity in the lags of the dependent variable. They introduced infinite order lag polynomials in the large N relationships between the unobserved factors and cross-sectional averages. More so, they focus on stationary coefficient heterogeneous panels including weakly exogenous regressors where the cross-sectional dimension (N) and the time series dimension (T) are large. The study centred on estimation and statistical inference of the mean coefficients, and deal with the small T bias of the estimators through bias correction methods.

Moon and Weidner (2013a and 2013b) proposed a Gaussian Quasi Maximum Likelihood Estimator (QMLE) to estimate dynamic panel data models and cross-sectionally dependent errors which assumed homogeneous coefficients in analysis. They emphasised that the estimator is not applicable to pooled data having lagged dependent variables of heterogeneous coefficients and lack ability to address Cross-Sectional (CS) dependence. Similarly, Bai (2009) developed the Interactive-Fixed Effects (IFE) estimator and allowed for cross-sectionally dependent errors but assumed homogeneous slopes. The analysis of Bai (2009) was extended by Song (2013) which allowed for coefficient heterogeneity as well as lagged dependent variable but gave results on the estimation of cross-section individual specific coefficients only. In addition, Pesaran (2006) proposed a mean group estimator of the mean coefficients and showed that CCE types estimators once increased with a sufficient number of lags and cross-sectional averages performed well even in the case of dynamic models with weakly exogenous regressors.

Pesaran and Smith(1995) showed that when the true model is heterogeneous and dynamic, the pooled estimators are inconsistent meanwhile an average estimator of heterogeneous dynamic panel data coefficient can result to consistent estimates as long as both N and T tend to infinity. They argued in favour of heterogeneous dynamic panel data estimators rather than static pooled estimators for panels with large N and T.

Robin Sickles and Tsionas (2013) proposed two panel data models with unobserved heterogeneous time-varying effects, first with individual-specific effects as random functions of time period while the second presented the time-varying effects with common factors whose number are not known and are firm-specific effects. These panel data models have two distinctive characteristics which are considered as a generalisation of conventional panel data models.

Firstly, the model examined by Bai (2009), Kneip *et al*. (2012), and Ahn *et al*. (2013) treated both the individual effects that are assumed to be heterogeneous among the units and time-varying effect as non-parametric.

Secondly, the Bayesian knowledge of random coefficient models developed by Swamy (1970), Swamy and Tavlas (1995) for two panel data models above will subjectively not assume a general functional form for all the individuals as the processes of its subjectivity. This may vary across the individuals and fixed parametric values to the estimated parameters which explained the relationship of functional form that may not be properly defined. Hence, a Bayesian technique overcome the computational intense and the theoretical complexity of nonparametric or semi-parametric regression approach, Yatchew(1998) and this prompt the need to depend on asymptotic theory for statistical inference, Koop and Poirier(2004).

Broeck *et al*. (1994) used a Bayesian method to panel data models with applications in stochastic frontier analysis under the composed error model. The establishment of a Bayesian approach to analyse model where the fixed and random effect are specified and applied through Gibbs Sampling way of simulations. Osiewalski and Steel(1998) applied Bayesian numerical integration approaches to perform the Bayesian analysis of the stochastic frontier model using cross-sectional data and panel data hence, the individual-specific effects are assumed to be time-invariant which is not appropriate in many economy settings. They suggested a Bayesian approach defining prior distribution for the unknown parameters in the model using Markov chain Monte Carlo to estimate the parameters of the resulting model for the individual choices. Blomquist and Westerlund (2013) suggested a method that is efficient to general forms of serial correlation and cross-sectional dependence. Also, the method for slope homogeneity in large-dimensional panel models with interactive fixed effects was developed by Su and Chen (2013).

Bayesian estimation through repeated iterative sampling from posterior densities make hierarchical modelling of panel data easier, whether random effects, time varying regressor effects or common factors, unstructured observation level errors or correlated in multivariate panel data.

Davidian and Giltinan (1995) considered parameters in the model as random under Bayesian Markov Chain Monte Carlo estimation, and are not integrated out ordinarily as often done in classical approaches. The marginal likelihood estimation or integrated approach used different maximisation methods to obtain parameter estimates of the model which may become unreliable or unachievable in difficult varying coefficient models, Tutz and Kauermann(2003) and Molenberghs and Verbeke (2004). Therefore, hierarchical Bayesian estimation of short-run slope coefficients proposed by Hsiao *et al* (1999) is found to have good small sample properties in the Monte Carlo study.

2.5 Review of Hierarchical Bayesian Estimation

Hierarchical is a multi-level model which is central to the analysis of Bayesian statistics for both theoretical and practical purposes; this allows us to incorporate richer and better information into the parameters of the model, Andrew (2006). Hierarchical Bayesian models, on the theoretical side, allow a more objective approach to statistical inference by estimating the slope coefficients of prior distributions from data rather than requiring them to be specified through subjective information, James and Stein(1960), Efron and Morris (1975) and Morris(1983). On the practical side, hierarchical Bayesian models are flexible devices for combining information and partial pooling of inferences, Carlin and Louis(2001) and Gelman *et al*, (2004). Hierarchical parameters in the model are random, specified with their own distribution and hyper priors. The implemented hierarchical Bayesian methods to the estimation of model take merit from modern computational methods for constructing a MCMC based simulation algorithm and employs data augmentation techniques for the latent variables of the model.

Hierarchical Bayesian estimation is an engine via MCMC techniques that generates draws from posterior distribution of the model parameters. The use of MCMC methods has ability to remove problems arising from many conventional analyses. The MCMC methods place a set of repeating calculations that in effect, simulations are drawn from distribution rather than deducing the analytic form of the known distribution of posterior. Criteria such as posterior means, numerical standard error, and highest posterior density intervals and so on are used to assess the performance of the posterior simulation techniques through the applications of Monte Carlo draws. Estimation of hierarchical Bayesian models with MCMC methods are helpful in that it gives estimates of all parameters in the model, including the estimates of model joined with individual-specific parameters. The use of MCMC techniques preside the functional interest of parameters in the model which are closely related to individual decisions making.

Allenby and Ginter (1995) reviewed the hierarchical Bayesian estimation with MCMC which gives freedom to explore the parameter estimate of high dimensional space. Missing data problems and hierarchical models are some of the problems that find solution in Bayesian method. The Bayesian method is useful in building model, performing estimation of model parameters and providing statistics inference for problems that are complicated and possibly not obvious in classical methods.
Bayesian methods are founded on the assumption of probability rules, functionalised as a degree of belief which is not found in classical statistics. Hierarchical Bayesian models deal with the possibility of variability in parameter across units by appropriating a model for the parameters. The "hierarchy" arises when the model for the parameters places "above" the model for the data. Bayesian models are hierarchical in that a prior of β is placed over the model for *y*. Simon (2008) investigates the statistical model notion nested as a hierarchy of stochastic relations displace all hierarchical modelling and also listed why hierarchical models are very agreeable to Bayesian analysis. Generally, the hierarchical Bayesian statistical model can be of the form;

$y_i \mid \beta_i \sim p(y_i \mid \beta_i)$ (Model for the data in group i= 1, . . . , I)

$\beta_i \mid \upsilon \sim p(\beta_i \mid \upsilon)$ (Between-group model or "prior" for the parameters β_i)

$\upsilon \sim p(\upsilon)$ (Prior for the *hyperparameters* υ),

The hierarchy are written from "bottom" to "top". The inferential problem will be to estimate the posterior density of all the parameters in the model, $\beta = (\beta_1, \beta_2, ..., \beta_i, \upsilon)'$ and possible marginal posterior densities for individual-specific elements of β that may be of interest using the property called conditional independence.

Research in quantitative analysis makes use of models with slope coefficients which are of focus in the analysis of hierarchical Bayesian models using probability distributions to quantify prior knowledge about the parameters not just the effect presence or absence but the information originated from the data to give a posterior distribution.

The hierarchical Bayesian model has value that lies in its capacity to characterise heterogeneity in options while maintaining its ability to study individual- specific effects of the parameter model. Browning and Carro, (2006) defined heterogeneity as the variations of factors that are known and important to individual agents when a particular decision is required. The nature, feature and determinant of heterogeneity have received much attention over the last decade, among the studies is the distribution of heterogeneity exhibited by a continuous and not a discrete distribution of heterogeneity, Allenby *et al*. (1998). Moreover, researchers have not correctly asserted the existence of a small number of homogeneous groups but have relevant implications for analysis connected with market segmentation. Hierarchical Bayesian approaches are being used to identify complicated variables that point to brand preferences, Yang *et al* (2003) while the new methods of dealing with individual heterogeneity in scale usage are originated from Rossi and Allenby(2001).

Rouder *et al*. (2007) demonstrated hierarchical Bayesian model, variability from nuisance sources such as individual agents and items are modelled simultaneously. The contribution to these models is not in mass data, and the results are processed in parameter estimates across items and individuals. In this case, the behaviour of this process of parameter estimates is not only studied across conditions, but across items and individuals as well, and provides activity of a process-model informed study of individual and item differences. Hence, hierarchical Bayesian models bridge the gap and account for inconvenience and nuisance variation that over-view the process-parameters into strength. Hierarchical models provide a means of exploring process varying across populations of items or individuals and making clearer view of process-parameters. Moreso, hierarchical linear models are models that extend regression analysis and analysis of variance to account for multiple sources of variance which are rampant in areas of social sciences as well as areas of psychology study, Raudenbush and Bryk(2002).

Adopting a Bayesian method has several advantages, one of the most pertinent for intellectual researchers is the building of hierarchical models which become easy to account for dispersion in real-world settings. If the available data provide only indirect information or small sample size about the parameters of study, the prior distribution becomes more essential. In many setting, models can be set up hierarchically, so that groups of parameters can be estimated from data shared prior distributions.

Silvio (2002) examined the use of Bayesian perspective in both fixed and random effects. He treated them as random parameters in stages of hierarchical model. The response variable and regressors are both distributed around a mean value on certain parameters model, these parameters later distributed around a mean value ascertained by other parameters called hyperparameters which are random in nature. Random effects estimation updates the prior distribution of the hyperparameters while fixed effects estimation updates the prior distribution of the parameters. Importantly, the distribution between fixed, random and mixed models is subjected to the distinction between different prior assigned to different stages of the hierarchy, Smith (1973).

Beck and Katz (1995, 1996) opined that the randomness resides in the parameters as the merit found in hierarchical Bayesian linear model which is not found in classical model of units. Hence, the difference between sampled units and fixed is no longer applicable, therefore making the approach yields similar results to the random coefficients model.

Stone and Springer (1965), Hill (1965), Tiao and Tan (1965) considered fully-Bayesian analyses of hierarchical linear models which remained a topic of applied interest and theoretical findings. Portnoy (1971), Box and Tiao (1973), Gelman et al. (2003), Carlin and Louis (1996) and Meng and Dyk(1999) provided Bayesian and non-Bayesian inference for hierarchical models which were compared by considering some different prior distributions for variance parameters.

Moreover, the principles of hierarchical prior distributions in the context of a definite class of models were discovered by Browne and Draper (2005). In the context of an expanded conditionally-conjugate family, hierarchical variance parameters expressed the prior distribution where noninformative prior distributions, including inverse-gamma and uniform families were examined. They emphasised a hierarchical model which demands for hyperparameters, where prior distributions of each parameter is defined and a proposed half-t model is demonstrated as a component in a variance parameters of hierarchical model and as a weakly-informative prior distribution which found a hierarchical method useful in Bayesian settings both in theory and practical.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter discusses the theoretical framework of a linear regression with panel data model, pooled model, individual effects model using a non-hierarchical prior and a hierarchical prior and random coefficients dynamic panel data model. Details of the hierarchical Bayesian computations are discussed. Also, the data generating scheme and the data simulation for sensitivity of prior information on posterior estimates are presented.

3.2 Theoretical Framework

3.2.1 Linear Regression Panel Data Model

The simple linear regression model with the panel data refers to the pooling of observations on cross-sectional data such as households, countries, and firms over several time periods. The traditional model for the panel data is the static panel data model. The basic framework for this static panel data is a regression model of the form

$$y_{it} = X_{kit}\beta_k + \varepsilon_{it} \tag{3.1}$$

Where y_{it} is the dependent variable, X_{kit} is the independent variables, $i = 1, 2, ..., N$, $t = 1, 2, ..., T$ and k = 1,2, . . . , K, while N, T and K are the cross-sectional, time series dimensions and number of unknown parameters respectively, *k*-vector of regression coefficients β including the ε_{it} is error term and intercept.

The asymptotic properties of the estimators in the traditional regression model in (3.1) are established under the following assumptions:

(i) $E(\varepsilon_{it} \mid X_{1t}, X_{2t}\ \ldots, X_{iT}) = 0$ (3.2)

(ii) $Var(\varepsilon_{it} \mid X_{1t}, X_{2t} \, . . . , X_{iT}) = \sigma_{\varepsilon}^{2}$ (3.3)

(iii) $Cov(\varepsilon_{it}, \varepsilon_{js} \mid X_{1t}, X_{2t}, \, . . . , X_{iT}) = 0$ if $i \neq j$ or $t \neq s$ (3.4)

If the remaining assumptions of the traditional model are met,in this form $E(\varepsilon_{it} \mid X_{it}) = 0$, homoscedasticity, independence across observations, *i*, and strict exogeneity of X_{1t}), then ordinary least squares is the efficient estimator and reliable inference can be made.

The model in equation (3.1) can be written more compactly in matrix form:

$$y_{it} = X_{it}\beta + \varepsilon_{it} \tag{3.5}$$

Defining the *NT* x 1 vectors:

$$y = \begin{bmatrix} y_{i,1} \\ y_{i,2} \\ . \\ . \\ . \\ y_{i,T} \end{bmatrix}, \text{ and } \varepsilon = \begin{bmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ . \\ . \\ . \\ \varepsilon_{i,T} \end{bmatrix}$$

And the NT x K matrix

$$X = \begin{bmatrix} X_{0i1} & X_{1i1} & X_{2i1} & . \; . \; . & X_{ki1} \\ X_{0i2} & X_{1i2} & X_{2i2} & . \; . \; . & X_{ki2} \\ . & . & & . & . \\ . & . & . & . \; . \; . & . \\ . & . & & . & . \\ X_{0iT} & X_{1iT} & X_{2iT} & . \; . \; . & X_{kiT} \end{bmatrix}, \qquad \text{K x 1 vector}, \; \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ \beta_k \end{bmatrix}$$

Implicitly X_{0it} is set to 1 which allow for an intercept. The K regression parameters in β provide information about each independent variable's unique relationship with the dependent variable.

3.2.2 Set-up of Bayesian Estimation of Panel Data Model

A. The Pooled Model

This model assumes that regression coefficients are identical for the entire individual (N) and time period (T) and that the common regression line is suitable for all units.

$$y_{it} = X_{it}\gamma + \varepsilon_{it} \tag{3.6}$$

Where y_{it} is the dependent variable, X'_{it} is row vector of explanatory variables, dimension *K*, $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, while N and T are the cross-sectional and time series dimensions respectively, γ denoteK-vector of regression coefficients, with the intercept,and ε_{it} is the error term which is normally distributed with mean zero and variance sigma squared that is, $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon}^2)$.

The error terms are independent over all individual and time period. The likelihood function form relies on assumptions made about the error terms.

The assumptions are:

(i) The ε_i has a multivariate Normal distribution with mean 0and variance h^{-1} that is, $\varepsilon_{it} \sim N(0,\ h^{-1})$

(ii) The ε_i and ε_j are independent of one another for $i \neq j$ that is, $\mathrm{E}(\varepsilon_i \varepsilon_j) = 0 \forall i \neq j$

(iii)The error term is statistically unrelated to the exogeneous variables: $E(X'_{it}\varepsilon_{it}) = 0$

(iv)The error term is uncorrelated with the lagged endogenous variable: $E(y_{i,t-1}\varepsilon_{it}) = 0$

These assumptions of errors are independent over entire cross-sectional and time periods making the model simply a linear regression model.

All the elements of explanatory variables are not random but fixed. Explanatory variables are random variables if they are independent of all elements of ε_j with a probability density function, $p(X_i|\theta)$ for θ is a vector of parameters that does not include γ and h.

Estimation of the linear regression model in equation (3.6) using Bayesian estimators can be executed via the following three steps, Simon (2009);

(i) Determine the likelihood function of the unknown parameters to be estimated.

(ii) Specify the prior distribution for all the unknown parameters in the model.

(iii) Define the posterior distribution of the parameters given the data.

The posterior distribution is proportional to the product of likelihood function and the prior distribution

The posterior function of equation (3.6) can be obtained as:

$$p(\gamma, h \mid y, X) \propto p(y \mid X, \gamma, h).p(\gamma).p(h)$$

The likelihood function of a Pooled Model

Suppose y_1 **. . . ,** y_N be a set of independently and identically distributed random variable of size N from continuous density function $f(y_i, \gamma)$ with an unknown parameter (γ). Then, the likelihood function of γ is given by

$$L(\gamma) = \prod_{i=1}^{n} f(y_i, \gamma) \tag{3.7}$$

It is noted from the linear regression model $y_i = X_i\gamma + \varepsilon_i$ in equation (3.5) that $\varepsilon_i \sim \boldsymbol{N(0, \sigma^2} I_N)$, we set $\text{var}(\varepsilon) = \sigma^2 I_N = h^{-1} I_N$ with error precision as $h = 1/\sigma^2$

In a matrix notation

$$\begin{pmatrix} \text{var}(\varepsilon_1) & \text{cov}(\varepsilon_1\varepsilon_2) & . & . & . & \text{cov}(\varepsilon_1\varepsilon_N) \\ \text{cov}(\varepsilon_1\varepsilon_2) & \text{var}(\varepsilon_2) & . & . & . & . \\ . & \text{cov}(\varepsilon_2\varepsilon_3) & . & . & . & . \\ . & . & . & . & . & \text{cov}(\varepsilon_{N-1}, \varepsilon_N) \\ \text{cov}(\varepsilon_1\varepsilon_N) & . & . & . & . & \text{var}(\varepsilon_N) \end{pmatrix} = \begin{pmatrix} h_1^{-1} & 0 & . & . & . & 0 \\ 0 & h_2^{-1} & . & . & . & . \\ . & . & h_3^{-1} & . & . & . \\ . & . & . & . & . & 0 \\ 0 & . & . & . & 0 & h_N^{-1} \end{pmatrix}$$

This is a matrix which contains the variances on the diagonal and covariances at the upper and lower diagonal, $\text{var}(\varepsilon) = h^{-1} I_N$ is a compact notation for $\text{var}(\varepsilon_i) = h^{-1}$ and $\text{cov}(\varepsilon_i\varepsilon_j) = 0$ for $i, j = 1,$ **. . . , *N*** and $i \neq j$

This shows that $y_i \sim N(X_i\hat{\gamma},\ h^{-1} I_n)$ for the regression mean is $X_i\gamma$ with random variable y_i as the data information, the expression for the likelihood density is denoted by $p(y \mid \gamma, h)$,

The likelihood function form relies upon assumptions made about the errors. Assumptions about ε_i and X_i ascertainthe likelihood functionform using the definition of the Normal density as:

$$P(y|\gamma,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y_i-X_i\gamma)^2}{2\sigma^2}\right] \tag{3.8}$$

Since i $\neq$ j, y_i and y_j are independent of one another, it follows that ε_i *and* ε_j are independent of one another since dependent variable is the function error terms.

By the definition of the multivariate Normal density, we can write the likelihood function as:

$$P(y|\gamma,h)=\frac{h^{\frac{NT}{2}}}{(2\pi)^{\frac{NT}{2}}}\left\{\exp\left[-\frac{h}{2}(y-X\gamma)'(y-X\gamma)\right]\right\} \tag{3.9}$$

we set $\sigma^2=h^{-1}$ with error precision as $h=\frac{1}{\sigma^2}$

Hence, the likelihood function can be obtained through

$$P(y|\gamma,\sigma^2)=\frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(y_i-X_i\gamma)^2\right] \tag{3.10}$$

It proves suitedto re-write the likelihood in a another way. The exponent $\sum_{i=1}^{N}(y_i-X_i\gamma)^2$ in equation (3.10) can be expressed as

$$\Sigma(y_i-X_i\gamma)^2=vs^2+(\gamma-\hat{\gamma})^2\sum_{i=1}^{N}X_i^2 \tag{3.11}$$

where

$$v=N-1$$

$$\hat{\gamma}=\frac{\Sigma X_i y_i}{\Sigma X_i^2}$$

$$s^2=\frac{\Sigma(y_i-X_i\hat{\gamma})^2}{v}$$

$$\therefore \Sigma(y_i-X_i\gamma)^2=\Sigma\left\{\left(y_i-X_i\hat{\gamma}\right)-X_i(\gamma-\hat{\gamma})\right\}^2$$

Substituting equation (3.11) into equation (3.10) we have,

$$P(y|\gamma,\sigma^2)=\frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\exp\left[-\frac{1}{2\sigma^2}(vs^2+(\gamma-\hat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right] \tag{3.12}$$

Substituting, $\sigma=h^{-1/2},\sigma^2=h^{-1},N=v+1$ into equation (3.12)

$$P(y/\gamma,h^{-1})=\frac{1}{(2\pi)^{\frac{N}{2}}h^{-N/2}}\exp\left[-\frac{1}{2h^{-1}}(vs^2+(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right] \quad (3.13)$$

$$P(y|\gamma,h^{-1})=\frac{h^{N/2}}{(2\pi)^{\frac{N}{2}}}\exp\left[-\frac{h}{2}(vs^2+(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right] \quad (3.14)$$

$N=v+1$

$$P(y|\gamma,h^{-1})=\frac{h^{\frac{v+1}{2}}}{(2\pi)^{\frac{N}{2}}}\exp\left[-\frac{h}{2}(vs^2+(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right] \quad (3.15)$$

$$P(y|\gamma,h^{-1})=\frac{h^{\frac{v}{2}}.h^{\frac{1}{2}}}{(2\pi)^{\frac{N}{2}}}\exp\left[-\frac{h}{2}(vs^2+(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right] \quad (3.16)$$

$$P(y|\gamma,h^{-1})=\frac{1}{(2\pi)^{\frac{N}{2}}}\left[\left(h^{\frac{v}{2}}\exp\frac{-hvs^2}{2}\right).\left(h^{\frac{1}{2}}\exp\frac{-h}{2}(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right)\right] \quad (3.17)$$

$$P(y|\gamma,h^{-1})=\frac{1}{(2\pi)^{\frac{N}{2}}}\left[\left(h^{\frac{v}{2}}\exp\frac{-hv}{2s^{-2}}\right).\left(h^{\frac{1}{2}}\exp\frac{-h}{2}(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right)\right] \quad (3.18)$$

$$P(y|\gamma,h^{-1})=\frac{1}{(2\pi)^{\frac{N}{2}}}\left[\left(h^{\frac{1}{2}}\exp\frac{-h}{2}(\gamma-\widehat{\gamma})^2\sum_{i=1}^{N}X_i^2)\right).\left(h^{\frac{v}{2}}\exp\frac{-hv}{2s^{-2}}\right)\right] \quad (3.19)$$

Using the matrix generalisation, the likelihood function can be written as above which indicates that the natural conjugate prior is Normal- Gamma density.

The Prior Distribution of a Pooled Model

We draw out a prior for γ conditional on h from equation (3.19) as stated below;

$$\gamma / h \sim N\left(\underline{\gamma}, h^{-1}\underline{V}\right) \tag{3.20}$$

And a prior for h given as

$$h \sim G\left(\underline{s^{-2}}, \underline{v}\right) \tag{3.21}$$

Then the joint priorhas Normal-Gamma distribution form given in notation form as:

$$\gamma,\ h \sim NG\left(\underline{\gamma},\ \underline{V},\ \underline{s^{-2}},\ \underline{v}\right) \tag{3.22}$$

Where $\underline{\gamma}$ is a k-vector containing the prior means for k regression slope coefficients $\gamma_1, \gamma_2, \ldots \gamma_k$ and $\underline{V}$ is a k x k positive definite prior covariance matrix.

The Posterior Inference of a Pooled Model

The posterior density is derived by multiplying the likelihood in (3.19) with prior density in (3.22)

$$\gamma, h \mid y \sim NG\left(\overline{\gamma},\ \overline{V}\ \overline{\mathrm{s}}^{-2},\ \overline{v}\right) \tag{3.23}$$

Where,

$$\overline{V} = (\underline{V}^{-1} + X'X)^{-1}$$

$$\overline{\gamma} = \left(\underline{V}^{-1}\underline{\gamma} + X'X\widehat{\gamma}\right)$$

$$\overline{v} = \underline{v} + N$$

and $\overline{s^{-2}}$ is defined through

$$\overline{vs}^2 = \underline{vs}^2 + vs^2 + (\widehat{\gamma} - \underline{\gamma})'\left[\underline{V} + (X'X)^{-1}\right]^{-1}(\widehat{\gamma} - \underline{\gamma})$$

v is the degree of freedom

$h = \dfrac{1}{\sigma^2}$ is the error precision

The mean γ is $E(\gamma \mid y) = \overline{\gamma}$ (3.24)

and

$$Var(\gamma / y) = \frac{\overline{vs}^2}{\overline{v} - 2}\overline{V} \tag{3.25}$$

The parameters of error precision are $E(h / y) = \overline{s}^{-2}$ (3.26)

and

$$Var(h/y) = \frac{2s^{-2}}{\overline{v}} \qquad (3.27)$$

The expressions in equations (3.24) to (3.27) describe the informative prior.

For non-informative prior, we assume that $\underline{v} = 0$ and $\underline{V}^{-1} = 0$ i.e $\underline{V} \to \infty$ where its posterior distribution has

$$\gamma, h \mid y \sim NG\left(\overline{\gamma},\ \overline{V}\ \overline{s}^{-2},\ \overline{v}\right) \qquad (3.28)$$

Where,

$$\overline{V} = (X'X)^{-1}$$

$$\overline{\gamma} = \widehat{\gamma}$$

$$\underline{v} = N$$

$$\overline{vs}^2 = vs^2$$

These formulasrequire only data information, and are equivalent to OLS quantities. Note that we use bars over parameters to represent parameters of a posterior density, and bars under parameters to represent parameters of the prior density.

A pooled model can be displayed in a graphical form as

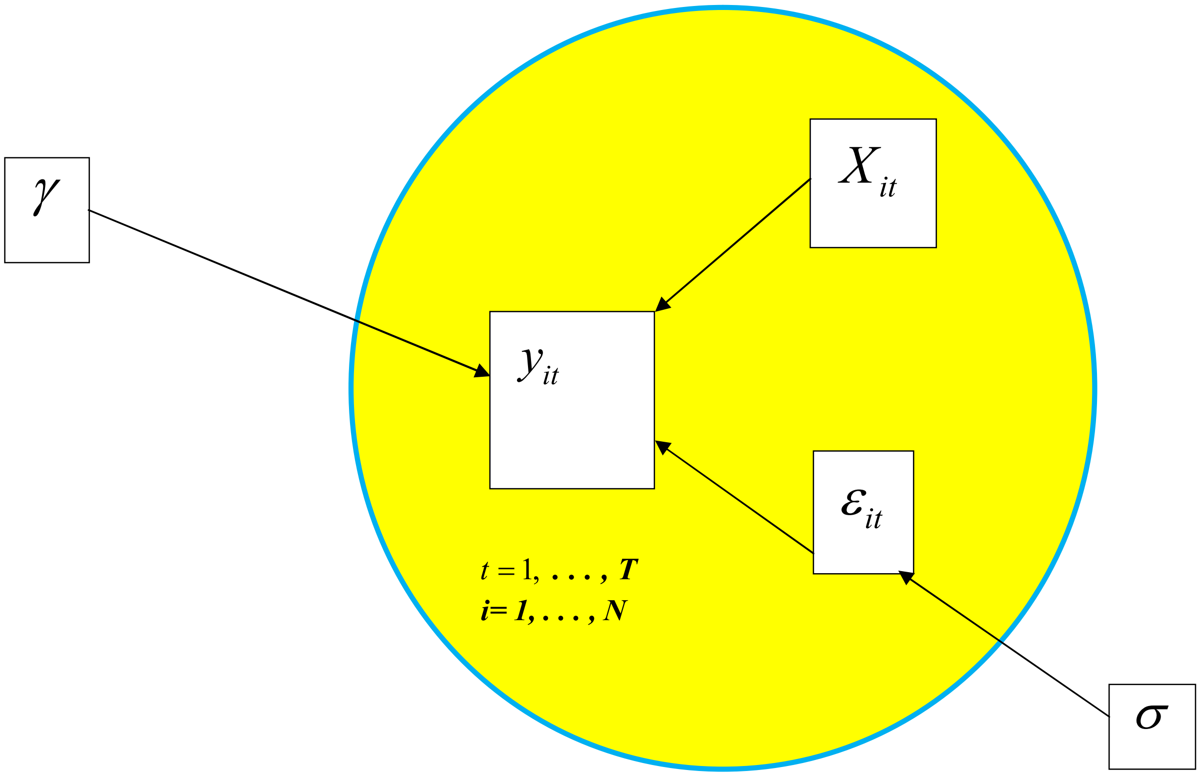

Figure 3.1: Graphical representation of a pooled model: $y_{it} = X_{it}\gamma + \varepsilon_{it}$, $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$

B. The Individual Effects Models

This is the model where the regression intercepts are permitted to differ across individuals (N). This assumes that the slope parameters are the same for all individuals but the intercepts are different across the individuals.It also assumes that all individuals (N) have regression line with the same slope but different intercepts,

The individual regression model is given as:

$$y_{it} = \theta_i + X_{it}\gamma + \varepsilon_{it} \tag{3.29}$$

Where y_{it} is the dependent variable, X_{it} is the independent variables, $i = 1,2,...,N$ and $t = 1,2,...,T$ for N and T are the cross-sectional and time series dimensions respectively, also γ describes the same parameter for all individuals, while θ_i are referred to as intercepts of the individual effects and ε_{it} is error term.

The Likelihood Function of Individual Effects Models

The likelihood function of equation (3.29) is based on the regression equation given as:

$$y_i = \theta_i l_T + \tilde{X}_i\tilde{\gamma} + \varepsilon_i \tag{3.30}$$

Where θ_i represents the intercepts of ith individual's regression equation and $\tilde{\gamma}$ represents the vector of regression parameters (which presumes to be the same for all individuals).

Equation (3.30) along with error assumptions given in the pooled model, a multivariate Normal density implies a likelihood function of the form

$$p(y \mid \theta, \tilde{\gamma}, h) = \prod_{i=1}^{N} \frac{h^{T/2}}{(2\pi)^{T/2}} \left\{ \exp\left[-\frac{h}{2}\left(y_i - \theta_i - \tilde{X}_i\tilde{\gamma}\right)'\left(y_i - \theta_i - \tilde{X}_i\tilde{\gamma}\right) \right] \right\} \tag{3.31}$$

Where $\theta = \left(\theta_1 ..., \theta_N\right)'$

The Prior Forms of Individual Effects Models

Bayesian inference may be contested because each parameter in the model requires that prior type must be specified for the model. These are subjectively chosen by the researcher depending on the parameter rigour of the model. However, we examine two kinds of prior which are commonly used and computationally simple.

(i) A Non- hierarchical prior

(ii) A Hierarchical prior

Individual Effects Models (A Non- Hierarchical Prior)

A non- hierarchical prior of a Bayesian analysis leads to a model which is similar to the fixed effects models of the classical approach.

To establish this fact equation (3.30) can be written as:

$$y = X^* \gamma^* + \varepsilon \tag{3.32}$$

Where y and ε is a NT x 1 vector and X^* is a NT x (N+ K -1) matrix, while γ^* is a $N+K$ vector given as

$$y = \begin{pmatrix} y_1 \\ . \\ . \\ . \\ y_N \end{pmatrix}, \quad X^* = \begin{pmatrix} l_T & 0_T & . & . & 0_T & \tilde{X}_1 \\ 0_T & l_T & . & . & . & \tilde{X}_2 \\ . & 0_T & . & . & . & . \\ . & . & . & . & 0_T & . \\ 0_T & . & . & . & l_T & \tilde{X}_N \end{pmatrix}, \quad \gamma^* = \begin{pmatrix} \theta_1 \\ . \\ . \\ \theta_N \\ \tilde{\gamma} \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ . \\ . \\ . \\ \varepsilon_N \end{pmatrix} \tag{3.33}$$

Where $\tilde{\gamma}$ is the slope coefficients of the individual without any intercepts and γ^* denote regression coefficients both slope and the intercepts, ε is the error term and y is the dependent variable. Also, X^* is a matrix which contains the independent variables attached to a matrix including a dummy variable for each individual.

The use of dummy variables is an attempt to specify a model with an error term that has zero mean, that is, effects are nuisance or incidental parameters which may distort a consistent estimation of the slope. Fixed effect models use dummy variables to account for class effects.

This model assumed independent Normal-Gamma prior and thus γ^* and h are priors independent of each other with

$$\gamma^* \sim N\left(\underline{\gamma}^*, \underline{V}\right) \tag{3.34}$$

and

$$h \sim G\left(\underline{s}^{-2},\ \underline{\nu}\right) \tag{3.35}$$

Individual Effects Models (A Hierarchical Prior)

The hierarchical prior is used to solve issues instigated by high dimensional parameter spaces. The equation in (3.30) isindividual effects model with a parameter space which contains $N + k$ parameters (that is, N intercepts in θ, k -1 slope coefficients in $\tilde{\gamma}$ together with the error precision, h). If T is small relative to N, the number of parameters is quite large relative to sample size. This indicates that a hierarchical prior might be suitable and such priors are indeed mostly used.

Such a hierarchical prior of a Bayesian analysis leads to a model which is similar to the classical random effects model.

A hierarchical prior assumes that, for i=1 , . . . , N

$$\theta_i \sim N(\mu_\theta, V_\theta) \tag{3.36}$$

With θ_i and θ_j being independent of one another for $i \neq j$. The hierarchical structures of the prior occur if μ_θ and V_θ are treated as unknown parameters which require their own prior information. We assume μ_θ and V_θ to be independent of one another along with their prior distribution

$$\mu_\theta \sim N\left(\underline{\mu}_\theta, \underline{\sigma^2}_\theta\right) \tag{3.37}$$

And

$$V_\theta^{-1} \sim G\left(\underline{V}_\theta^{-1}, \underline{\nu}_\theta\right) \tag{3.38}$$

For the other parameters, we incorporate a non-hierarchical prior of the independent Normal-Gamma prior. Thus

$$\tilde{\gamma} \sim N(\underline{\gamma},\ \underline{V}_\gamma) \tag{3.39}$$

and

$$h \sim G(\underline{s}^{-2},\ \underline{\nu}) \tag{3.40}$$

The Posterior Forms of Individual Effects Models

Posterior Inference Individual Effects Models (A Non- Hierarchical Prior)

The posterior inference can be executed using Gibbs sampler method which takes sequential draw from

$$\gamma^* / y, h \sim N\left(\overline{\gamma}^*, \overline{V}\right) \tag{3.41}$$

and

$$h / y, \gamma^* \sim G\left(\overline{s}^{-2}, \overline{v}\right) \tag{3.42}$$

where

$$\overline{V} = \left(\underline{V}^{-1} + hX^{*'}X^*\right)^{-1}$$

$$\overline{\gamma}^* = \overline{V}\left(\underline{V}^{-1}\underline{\gamma}^* + hX^{*'}y\right)$$

$$\overline{v} = NT + \underline{v}$$

and

$$\overline{s}^{-2} = \frac{\sum_{i=1}^{N}\left(y_i - \theta_i l_T - \tilde{X}_i\tilde{\gamma}\right)'\left(y_i - \theta_i l_T - \tilde{X}_i\tilde{\gamma}\right) + \underline{vs^2}}{\overline{v}}$$

The convergence and degree of approximation implied in the Gibbs sampler can be estimated using MCMC diagnostics described in Koop (2003).

The individual effects models (a non- hierarchical prior) can be displayed in a graphical form as

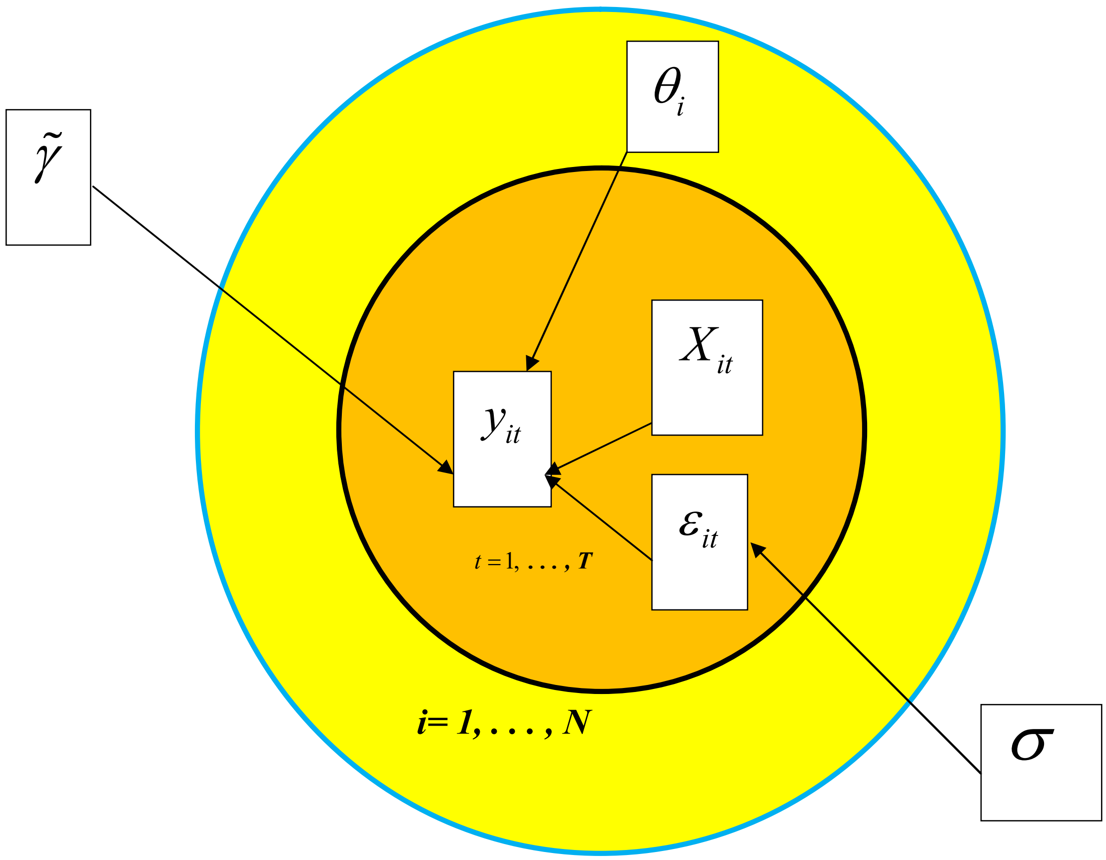

Figure 3.2: Graphical representation of the individual effects models (a non- hierarchical prior): $y_i = \theta_i l_T + \tilde{X}_i \tilde{\gamma} + \varepsilon_i$, $i = 1, 2, ..., N$

Individual Effects Models (A Hierarchical Prior) Posterior Inference

The derivation involves multiplying the likelihood with prior and then investigating the outcome for each of $\tilde{\gamma}$, h, θ, μ_θ and V_θ to find the kernels of each conditional posterior distribution under the hierarchical prior given in (3.36) through (3.40). The posterior simulation used Gibbs sampler algorithm which are drawn sequentially from posterior conditionals. The appropriate posterior distributions for $\tilde{\gamma}$ and h, conditional on θ, are derived in the same manner as those in equation (3.8) through (3.19) with independent Normal-Gamma prior

$$\tilde{\gamma} / y,\ h,\ \theta,\ \mu_\theta,\ V_\theta \sim N(\overline{\gamma},\ \overline{V}_\gamma) \tag{3.43}$$

and

$$h / y,\ \tilde{\gamma},\ \theta,\ \mu_\theta,\ V_\theta \sim G\left(\overline{s}^{-2},\ \overline{v}\right) \tag{3.44}$$

Where

$$\overline{V_\gamma} = \left(\underline{V}_\gamma^{-1} + h\sum_{i=1}^{N} \tilde{X}'\tilde{X}_i \right)^{-1}$$

$$\overline{\gamma} = \overline{V}\left(\underline{V}_\gamma^{-1}\underline{\gamma} + h\sum_{i=1}^{N} \tilde{X}'\left[y_i - \theta_i l_T \right] \right)$$

$$\overline{v} = NT + \underline{v}$$

and

$$\overline{s}^2 = \frac{\sum_{i=1}^{N}\left(y_i - \theta_i l_T - X_i'\tilde{\gamma} \right)'\left(y_i - \theta_i l_T - X_i'\tilde{\gamma} \right) + \underline{vs^2}}{\overline{v}}$$

The conditional posteriors for each θ_i is independent of θ_j for $i \neq j$ and is given by

$$\theta_i \mid y,\ \tilde{\gamma},\ \theta,\ \mu_\theta,\ V_\theta \sim N\left(\overline{\theta}_i,\ \overline{V}_i\right) \tag{3.45}$$

where

$$\overline{V}_i = \frac{V_\theta\ h^{-1}}{TV_\theta + h^{-1}}$$

And

$$\overline{\theta_i} = \frac{V_\theta(y_i - \tilde{X}_i\tilde{\gamma})' l_T + h^{-1}\mu_\theta}{(TV_\theta + h^{-1})}$$

The conditional posteriors for the hierarchical parameters, μ_θ and V_θ are

$$\mu_\theta \mid y,\ \tilde{\gamma},\ h,\ \theta,\ V_\theta \sim N(\overline{\mu}_\theta,\ \overline{\sigma}_\theta^2) \quad (3.46)$$

and

$$V_\theta^{-1} \mid y,\ \tilde{\gamma},\ h,\ \theta,\ \mu_\theta \sim G(\overline{V_\theta}^{-1}, \overline{v}_\theta) \quad (3.47)$$

where

$$\overline{\sigma}_\theta^2 = \frac{V_\theta \underline{\sigma}^2{}_\theta}{V_\theta + N\underline{\sigma}^2{}_\theta}$$

$$\overline{\mu}_\theta = \frac{V_\theta \underline{\mu}_\theta + \underline{\sigma}^2{}_\theta \sum_{i=1}^{N} \theta_i}{V_\theta + N\underline{\sigma}^2{}_\theta}$$

$$\overline{v}_\theta = \underline{v}_\theta + N$$

and

$$\overline{V}_\theta = \frac{\sum_{i=1}^{N} (\theta_i - \mu_\theta)^2 + \underline{V}_\theta \underline{v}_\theta}{\overline{v}_\theta}$$

The Gibbs sampler for the posterior stimulator involves only random number generation from Normal-Gamma distributions of equation (3.43) through (3.47).

Both the non-hierarchical and hierarchical priors permit for every individual to have uncommon intercept. However, more structure is placed on parameter through the hierarchical prior in that it assumes all intercepts are generated from the same distribution. This extra structure, if consistent with patterns in the data allows for more precise estimation.

The individual effects models (a hierarchical prior) can be displayed in a graphical form as

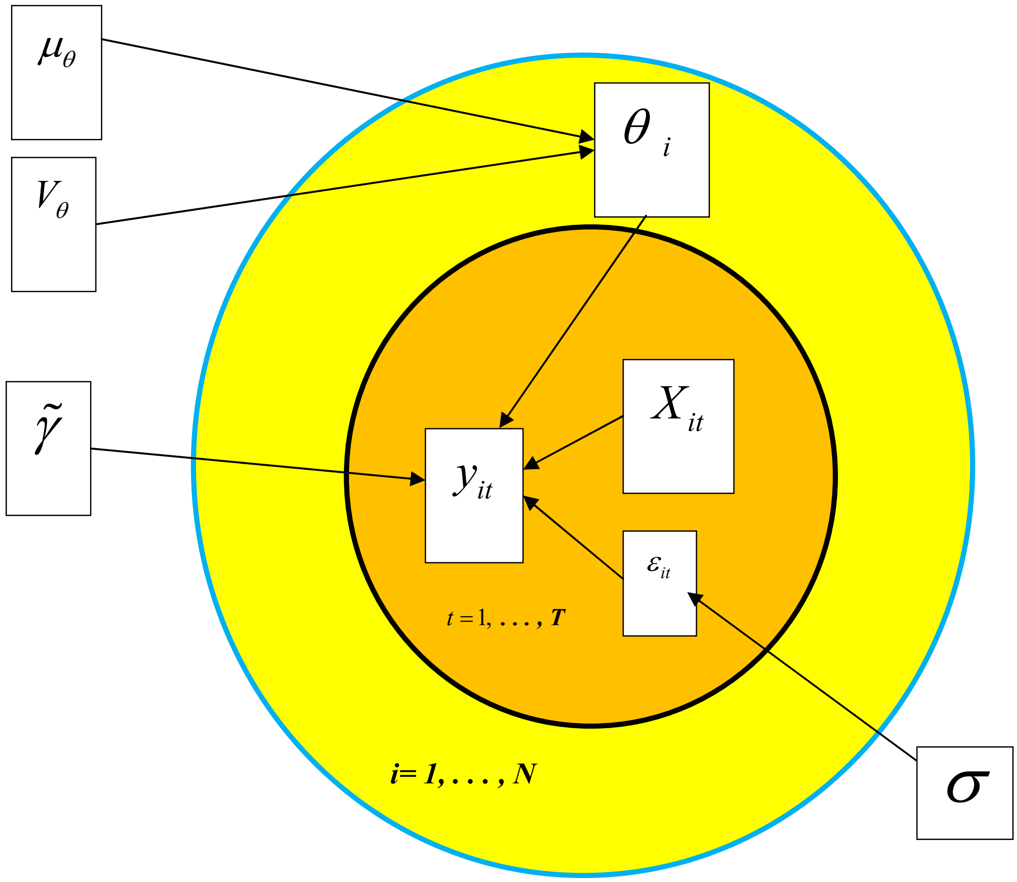

Figure 3.3: Graphical representation of the individual effects models (hierarchical prior): $y_i = \theta_i l_T + \tilde{X}_i \tilde{\gamma} + \varepsilon_i$, $i = 1, 2, \ldots, N$. Hierarchically, $\theta_i \sim N(\mu_\theta, V_\theta)$

C. Random-Coefficients Model

The random coefficients model allows for every individual β_i to be different. It uses a K-vector of regression coefficients and intercept denoted by

$$y_i = X_i\beta_i + \varepsilon_i \tag{3.48}$$

Where y is the response variable, X' is row vector of independent variables with dimension k, $i = 1, 2, ..., N$ and the whole model contains $Nk + 1$ parameters, that is k regression coefficients for each of N individuals together with the error precision, h. β is the k-vector of regression coefficients, with the intercept,and ε is the error term, independently and identically distributed with mean zero and variance sigma squared that is $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$.

Hence, the pooled model assumes that the common regression line is suitable for all individuals, while the individual effects models assume all individuals had regression lines with the common slopes, but possibly with different intercepts. Meanwhile, the random-coefficients model assumes all regression coefficients vary across individuals (both slope coefficients and intercepts). The random-coefficients model are practiced when the errors are cross-sectional, heteroscedastic and contemporaneously correlated with the first-order autoregression of the dependent variables.

D. Random Coefficients Dynamic Panel Data Model

Many economic relationships are dynamic in nature and one of the benefits of pooling data is that they permit the researcher to better understand the dynamics of modification. The dynamic relationships are characterised by the presence of a lagged dependent variable among the explanatory variables and individual effects describing the heterogeneity amidst the individuals in the model. Random coefficients model enables researchers to study the existence of heterogeneity (individual differences) among the regression parameters. Failure to account for the heterogeneity produces inconsistent estimates of the mean autoregressive coefficient, for panel of large N and large T, Smith (1995).

A random coefficient dynamic panel data model can be described as:

$$y_{it} = \delta_i y_{i,t-1} + \sum_{k=0}^{K} X_{kit}\beta_{ki} + \varepsilon_{it} \tag{3.49}$$

$i = 1, ..., N$, $t = 1, ..., T$ and k =0, 1, 2

$$y_{it} = \delta_i y_{i,t-1} + \beta_{0i} + X_{1it}\beta_{1i} + X_{2it}\beta_{2i} + \varepsilon_{it} \quad (3.50)$$

Now, the above model is generalised as

$$y_{it} = X_{it}^{*}\gamma_i + \varepsilon_{it} \quad (3.51)$$

Where

$$X_{it}^{*} = (X_{it} y_{i,t-1})'$$

And

$$\gamma_i = (\beta_i' \delta_i)'$$

Subscript $'i'$ indicates that the marginal effect of X^{*} on y varies across the units which suggests the presence of heterogeneity, that is, non-constant error variance.

The model in equation (3.50) is writing in matrix form

Defining the *NT* x 1 vector:

$$y = \begin{bmatrix} y_{i,1} \\ y_{i,2} \\ . \\ . \\ . \\ y_{i,T} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ . \\ . \\ . \\ \varepsilon_{i,T} \end{bmatrix}$$

And

The NT x NK matrix $X^{*} = \begin{bmatrix} y_{i,0} & X_{0i1} & X_{1i1} & X_{2i1} & . & . & . & X_{ki1} \\ y_{i,1} & X_{0i2} & X_{1i2} & X_{2i2} & . & . & . & X_{ki2} \\ . & . & . & & & . & & \\ . & . & . & & & . & & \\ . & . & . & & & . & & \\ y_{i,T-1} & X_{0iT} & X_{1iT} & X_{2iT} & . & . & . & X_{kiT} \end{bmatrix},$

$$NK \text{ x1 vector } \gamma = \begin{bmatrix} \delta_i \\ \beta_{0i} \\ . \\ . \\ . \\ \beta_{ki} \end{bmatrix}$$

Where y is $(NT\times 1)$ vector of dependent variable, X^* is a $(NT\times NK)$ matrix of unit specific regressors, γ is $(NK\times 1)$ vector of parameters, and ε is $(NT\times 1)$ vector of error terms that has a well-defined probabilistic properties. The disturbancetermsmay well represent all those factors that affect response variable but arenot taken into account explicitly. Since y_{it} is a function of ε_i, it follows immediately that $y_{i,t-1}$ is also a function of ε_i, Baltagi (2005). Therefore $y_{i,t-1}$, a right-hand regressor in equation (3.50) is correlated with the error term which render OLS estimator biased and inconsistent.

A random coefficient dynamic panel data model set-up in equation (3.51) requires likelihood function, a specification of the prior distributions over the parameters (γ_i and h) and computation on the posterior distribution using Bayesian learning process:

Likelihood Function of the Random Coefficient Dynamic Panel Data Model

The pattern of the likelihood function relies upon assumptions made about the errors. Assumptions about ε_i and X_i^* determine the pattern of the likelihood function using the definition of the Normal density as:

$$P(y\,|\,\gamma,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y_i - X_i^*\gamma_i)^2}{2\sigma^2} \right] \tag{3.52}$$

Since i $\neq$ j, ε_i *and* ε_j are independent of one another, it follows that y_i and y_j are also independent of one another. The regression mean of Normal density in equation (3.52) is $X_i^*\gamma_i$ with random variable y_i as the data information. This shows that $y \sim N(X_i^*\hat{\gamma}_i,\, h^{-1}I_n)$, the expression for the likelihood density denoted by $p(y\,|\,\gamma,h)$, using the definition of the multivariate Normal density, we can write the likelihood function as:

$$p(y\,|\,\gamma,h)=\frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}\left\{\exp[-\frac{h}{2}(y_i-X_i^*\gamma_i)'(y_i-X_i^*\gamma_i)]\right\} \tag{3.53}$$

Hence, the likelihood function of equation (3.52) becomes

$$P(y\,|\,\gamma,\sigma^2)=\frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(y_i-X_i^*\gamma_i)^2\right] \tag{3.54}$$

Substitute, $h={}^{1}\!/_{\sigma^2} \Rightarrow \sigma^2=h^{-1}$

$$p(y\,|\,\gamma,\sigma^2)=\frac{1}{(2\pi)^{N/2}h^{-N/2}}\exp\{-\frac{1}{2h^{-1}}\sum(y_i-X_i^*\gamma_i)'(y_i-X_i^*\gamma_i)\} \tag{3.55}$$

Where,

$$\hat{\gamma}_i=(X_i^{*\prime}X_i^*)^{-1}X_i^{*\prime}y_i$$

and

$$s^2=\frac{(y_i-X_i^*\gamma_i)'(y_i-X_i^*\gamma_i)}{v}\ .$$

Also,

$$\Sigma(y_i-X_i^*\hat{\gamma}_i)'(y_i-X_i^*\hat{\gamma}_i)=\ vs^2+(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i) \tag{3.56}$$

$$\therefore\ P(y\,|\,\gamma,\sigma^2)=\frac{h^{N/2}}{(2\pi)^{N/2}}\exp\{-\frac{h}{2}[vs^2+(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\} \tag{3.57}$$

For $v=N-k, N=v+k$

$$P(y\,|\,\gamma,\sigma^2)=\frac{h^{v+k/2}}{(2\pi)^{N/2}}\exp\{-\frac{h}{2}[vs^2+(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\} \tag{3.58}$$

$$P(y\,|\,\gamma,\sigma^2)\ =\frac{h^{v/2}h^{k/2}}{(2\pi)^{N/2}}\exp\{-\frac{h}{2}[vs^2+(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\} \tag{3.59}$$

$$P(y\,|\,\gamma,\sigma^2)=\frac{1}{(2\pi)^{N/2}}\left(\{h^{k/2}\exp[-\frac{h}{2}(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\}\{h^{v/2}\exp-\frac{h}{2}vs^2\}\right) \tag{3.60}$$

$$P(y\,|\,\gamma,\sigma^2)=\frac{1}{(2\pi)^{N/2}}\left(\{h^{k/2}\exp[-\frac{h}{2}(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\}\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\}\right) \tag{3.61}$$

The quantity $\{h^{k/2}\exp[-\frac{h}{2}(\gamma_i-\hat{\gamma}_i)'X_i^{*\prime}X_i^*(\gamma_i-\hat{\gamma}_i)]\}$ in equation (3.61) resembles the kernel of the multivariate normal density while $\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\}$ also looks like the kernel of the gamma density. These results simply suggest a normal-gamma prior for the likelihood function, Koop(2003).

$$P(y\,|\,\gamma,\sigma^2)=\frac{1}{(2\pi)^{N/2}}\left(\{h^{k/2}\exp[-\frac{h}{2}(\gamma_i-\hat{\gamma}_i)'V^{-1}(\gamma_i-\hat{\gamma}_i)]\}\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\}\right) \quad (3.62)$$

Where $V=(X^{*\prime}X^*)^{-1}$ is simply the prior covariance matrix of γ_i

$$P(y\,|\,\gamma,\sigma^2)=$$

$$\frac{1}{(2\pi)^{v/2}.(2\pi)^{k/2}}|V|^{-\frac{1}{2}}\left(\{\exp[-\frac{1}{2}(\gamma_i-\hat{\gamma}_i)'V^{-1}(\gamma_i-\hat{\gamma}_i)]\}\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\}\right) \quad (3.63)$$

The independent normal-gamma prior has posterior conditional densities that are non-analytical in nature. Different from the natural conjugate normal-gamma prior, which can be denoted as $p(\gamma_i,h)\sim f_{NG}(\gamma_i,h\,|\,\underline{\gamma}_i,\underline{V}_i,\underline{s}^{-2},\underline{v})$, because of the theorem of independence, we cannot join the independent prior together, that is, $p(\gamma_i,h)=p(\gamma_i).p(h)$, Feller (1971).

Prior Distribution of the Random Coefficient Dynamic Panel Data Model

A prior is the information we have about a particular study before seeing the data, we denote the independent prior by $p(\gamma_i,h)$.Therefore, from the law of independent random variables we have that $p(\gamma_i,h)=p(\gamma_i).p(h)$

From equation (3.62)

$p(\gamma_i)\sim Normal$

And

$p(h)\sim Gamma$

$$p(\gamma_i)=\frac{1}{(2\pi)^{\frac{k}{2}}}|V_i|^{-\frac{1}{2}}\exp[-\frac{1}{2}(\gamma_i-\underline{\gamma}_i)'\underline{V}_i^{-1}(\gamma_i-\underline{\gamma}_i)] \quad (3.64)$$

and

$$p(h) = C_G^{-1} h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \tag{3.65}$$

Where, C_G^{-1} is an integrating constant, It is deduced that: $E[\gamma_i \mid y_i] = \underline{\gamma}_i$ is the prior mean of γ_i and $Var(\gamma_i \mid h) = \underline{V}_i$ is the prior covariance matrix of γ_i with the mean of h, as $\underline{s}^{-2}$ and $\underline{v}$ degree of freedom.

Posterior Distribution of the Random Coefficient Dynamic Panel Data Model

A posterior can be described as the product of the likelihood and the prior which implies the information obtained about the parameters after seeing the data. Applying some mathematical methods, the posterior can be an independent, a conjugate or not taking a common distribution form.

It is usually denoted by $p(\gamma_i, h \mid y_i)$.

Mathematically, using $p(\gamma_i,\ h \mid y) = p(y_i \mid \gamma_i, h).p(\gamma_i).p(h)$

but note that $p(\gamma_i, h \mid y_i) \neq p(\gamma_i \mid y_i, h) \cdot p(h \mid y_i, \gamma_i)$ which makes the prior an independent normal-gamma prior and the posterior conditional densities are non-analytical forms. Then, the posterior is proportional to the likelihood times the prior. Hence, if wemultiply equation (3.53), (3.64) and (3.65), we will obtain

$$P(\gamma_i, h \mid y) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \exp\left[-\frac{h}{2}\left(y_i - X_i^* \gamma_i\right)' \left(y_i - X_i^* \gamma_i\right)\right] \cdot \frac{1}{(2\pi)^{\frac{k}{2}}} \left|V_i\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\gamma_i - \underline{\gamma}_i\right)' \underline{V}_i^{-1} \left(\gamma_i - \underline{\gamma}_i\right)\right] \cdot C_G^{-1} h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \tag{3.66}$$

$$p(\gamma_i, h \mid y) \propto \exp[-\frac{1}{2}\{h(y_i - X_i^* \gamma_i)'(y_i - X_i^* \gamma_i) + (\gamma_i - \underline{\gamma_i})' \underline{V}_i^{-1} (\gamma_i - \underline{\gamma_i})\}] . h^{\frac{N+\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \tag{3.67}$$

This joint posterior density for γ_i and h does not yield any well-known distributional form; so it cannot be solved analytically but only through a posterior simulation techniques.

The Simplification of the Posterior Distribution in a Matrix Multiplication

$$h\left(y_i - X_i^*\gamma_i\right)'(y_i - X_i^*\gamma_i) + (\gamma_i - \underline{\gamma}_i)'\underline{V}_i^{-1}(\gamma_i - \underline{\gamma}_i) \tag{3.68}$$

$$= h\left(y_i'y_i - 2\gamma_i'X_i^{*\prime}y_i + \gamma_i X_i^{*\prime}X_i^*\gamma_i\right) + (\gamma_i - \underline{\gamma}_i)'\underline{V}_i^{-1}(\gamma_i - \underline{\gamma}_i)$$

$$= hy_i'y_i - 2h\gamma_i'X_i^{*\prime}y_i + h\gamma_i'X_i^{*\prime}X_i^*\gamma_i + \gamma_i'\underline{V}_i^{-1}\gamma_i - 2\gamma'_i\underline{V}_i^{-1}\underline{\gamma}_i + \underline{\gamma}'_i\underline{V}_i^{-1}\underline{\gamma}$$

$$= hy_i'y_i + \underline{\gamma}'_i\underline{V}_i^{-1}\underline{\gamma}_i + \gamma_i'(\underline{V}_i^{-1} + hX_i^{*\prime}X_i^*)\gamma_i - 2\gamma_i'(hX_i^{*\prime}y_i + \underline{V}_i^{-1}\underline{\gamma}_i) \tag{3.69}$$

Let $\overline{V}_i = (\underline{V}_i^{-1} + hX_i^{*\prime}X_i^*)^{-1}$

$$\Rightarrow \overline{V}_i^{-1} = (\underline{V}_i^{-1} + hX_i^{*\prime}X_i^*) \text{ and } \overline{\gamma}_i = \overline{V}_i(hX_i^{*\prime}y_i + \underline{V}_i^{-1}\underline{\gamma}_i)$$

Hence, substituting back to equation (3.63) we have:

$$= hy_i'y_i + \underline{\gamma}_i'\underline{V}_i^{-1}\underline{\gamma}_i + \gamma_i'\overline{V}_i^{-1}\gamma_i - 2\gamma_i'\overline{V}_i^{-1}\overline{\gamma}_i \tag{3.70}$$

carrying out a simple mathematical assumption by including $-\overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$ and $+\overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$ into the equation (3.70) does not change anything in the equation but only making us realise the desired result we want, we have;

$$= hy_i'y_i + \underline{\gamma}_i'\underline{V}_i^{-1}\underline{\gamma}_i + \gamma_i'\overline{V}_i^{-1}\gamma_i - 2\gamma_i'\overline{V}_i^{-1}\overline{\gamma}_i - \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i + \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$$

$$= hy_i'y_i + \underline{\gamma}_i'\underline{V}_i^{-1}\underline{\gamma}_i - \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i + \gamma_i'\overline{V}_i^{-1}\gamma_i - 2\gamma_i'\overline{V}_i^{-1}\overline{\gamma}_i + \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$$

Let, $\omega = hy_i'y_i + \underline{\gamma}_i'\underline{V}_i^{-1}\underline{\gamma}_i - \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$ and

$$(\gamma_i - \overline{\gamma}_i)'\overline{V}_i^{-1}(\gamma_i - \overline{\gamma}_i) = \gamma_i'\overline{V}_i^{-1}\gamma_i - 2\gamma_i'\overline{V}_i^{-1}\overline{\gamma}_i + \overline{\gamma}_i'\overline{V}_i^{-1}\overline{\gamma}_i$$

$$\Rightarrow h(y_i - \gamma_i X_i^*)'(y_i - \gamma_i X_i^*) + (\gamma_i - \underline{\gamma}_i)'\underline{V}_i^{-1}(\gamma_i - \underline{\gamma}_i) = (\gamma_i - \overline{\gamma}_i)'\overline{V}_i^{-1}(\gamma_i - \overline{\gamma}_i) + \omega \qquad (3.71)$$

Substituting the equation (3.71) for the expression in equation (3.67), we obtain

$$P(\gamma_i, h \mid y) \propto \exp\left[-\frac{1}{2}\{(\gamma_i - \overline{\gamma}_i)'\overline{V}_i^{-1}(\gamma_i - \overline{\gamma}_i)\}\right] \cdot \exp[-\frac{1}{2}\omega]h^{\frac{N+\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) (3.72)$$

By ignoring the terms that do not involve γ_i in equation (3.72) we obtain,

$$P(\gamma_i \mid y, h) \propto \exp\left[-\frac{1}{2}\{(\gamma_i - \overline{\gamma}_i)'\overline{V}_i^{-1}(\gamma_i - \overline{\gamma}_i)\}\right] \qquad (3.73)$$

This is the kernel of a multivariate Normal density. In other words

$$\gamma_i \mid y, h \sim N(\overline{\gamma}_i, \overline{V}_i) \qquad (3.74)$$

Where,

$$\overline{V}_i = (\underline{V}_i^{-1} + hX_i^{*\prime}X_i^*)^{-1}$$

and

$$\overline{\gamma}_i = \overline{V}_i(hX_i^{*\prime}y_i + \underline{V}_i^{-1}\underline{\gamma}_i)$$

Similarly, by treating (3.67) as a function of h ignoring terms that do not involve h we can obtain

$$P(h \mid y, \gamma_i) \propto h^{\frac{N+\underline{v}-2}{2}} \exp[-\frac{h}{2}\{(y_i - X_i^*\gamma_i)'(y_i - X_i^*\gamma_i) + \underline{v}\underline{s}^2\}] \qquad (3.75)$$

By comparing (3.75) with the definition of the Gamma density, it has a posterior distribution as:

$$h \mid y, \gamma_i \sim G(\overline{s}^{-2}, \overline{v}) \qquad (3.76)$$

The posterior conditional densities in equation (3.72) yield density that is non-analytical in nature. The formulae of equations (3.73) and (3.75) look the same to those of the conjugate normal-gamma priors, but it does not relate directly to the posterior of interest since $P(\gamma_i, h \mid y) \neq P(\gamma_i \mid y, h) \cdot P(h \mid y, \gamma_i)$.

Therefore, the conditional posteriors in equation (3.74) and (3.76) do not directly describe everything about the posterior, $P(\gamma_i, h \mid y)$.

However, there is a posterior simulator called the Gibbs Sampler which makes use of the conditional posteriors like (3.74) and (3.76) to create random draws $\gamma^{(s)}$ and $h^{(s)}$ for

$s = 1,2,...,S$ which can be averaged to produce parameter estimates of the posterior properties just as the Monte Carlo integration.

The Gibbs sampling algorithm used in this study generates a sequence of random samples from the conditional posterior distributions of each parameter, in turn conditional on the current values of the other parameters, and thus generate a sequence of samples that constitute a Markov Chain, where the stationary distribution of that Markov chain is just the desired joint distribution of all the parameters.

Hierarchical Prior for the Random Coefficients Dynamic Panel Data Model

In the case of fully hierarchical priors, a Markov Chain Monte Carlo method (the Gibbs sampler) is employed to calculate posterior distributions. Such an approach is particularly useful in our framework since it exploits the recursive characteristics of the posterior distribution.

Assuming γ_i for $i = 1,...,N$ are independent draws from a normal distribution and thus,

$$\gamma_i \sim N(\mu_\gamma,\ V_\gamma) \tag{3.77}$$

With γ_i and γ_j being independent of one another for $i \neq j$. The hierarchical structures of the prior occur if μ_γ and V_γ are treated as unknown parameters which require their own prior information. We assumed that μ_γ and V_γ to be independent of one another with prior distribution as

$$\mu_\gamma \sim \boldsymbol{N}\left(\underline{\mu}_\gamma, \underline{\Sigma}_\gamma\right) \tag{3.78}$$

and

$$V_\gamma \sim \boldsymbol{W}\left(\underline{\boldsymbol{v}}_\gamma, \underline{V}_\gamma^{-1}\right) \tag{3.79}$$

The Wishart distribution is a matrix generalisation of the Gamma distribution. Defining the covariance matrix of a vector which includes the covariances appears at the upper and lower caes of the diagonal element while the variances of all the elements occupy the diagonal.

$\underline{\mu}_\gamma$ is now a *k*-vector containing the prior means for the *k* regression coefficients, and $\underline{V}_\gamma$ is now a $K \times K$ positive definite prior covariance matrix.

For the error precision, using the familiar Gamma prior:

$$h \sim G(\underline{s}^{-2}, \underline{v}) \tag{3.80}$$

The conditional posteriors for the $\gamma_i^{'}s$ are independent of one another, for $i = 1,...,N$, with

$$\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i) \tag{3.81}$$

Where

$$\overline{V}_i = (hX_i^{*\prime} X_i^* + V_\gamma^{-1})'$$

and

$$\overline{\gamma}_i = \overline{V}_i (hX_i^{*\prime} y_i + V_\gamma^{-1} \mu_\gamma)$$

For the hierarchical parameters, μ_γ and V_γ, the relevant posterior conditionals are

$$\mu_\gamma \mid y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma) \tag{3.82}$$

and

$$V_\gamma^{-1} \mid y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \ \overline{V}_\gamma\right]^{-1}) \tag{3.83}$$

Where

$$\overline{\Sigma}_\gamma = \left(NV_\gamma^{-1} + \underline{\Sigma}_\gamma^{-1}\right)^{-1}$$

$$\overline{\mu}_\gamma = \overline{\Sigma}_\gamma \left(V_\gamma^{-1} \sum_{i=1}^{N} \gamma_i + \underline{\Sigma}_\gamma^{-1} \underline{\mu}_\gamma \right)$$

$$\overline{v}_\gamma = N + \underline{v}_\gamma$$

$$\overline{V}_\gamma = \sum_{i=1}^{N} (\gamma_i - \mu_\gamma)(\gamma_i - \mu_\gamma)' + \underline{V}_\gamma$$

Since $\sum_{i=1}^{N} \gamma_i$ is the k-vector containing the sums of the elements of γ_i.

The posterior conditional for the error precision has the form:

$$h \mid y, \gamma, V_\gamma, \mu_\gamma \sim G(\overline{s}^{-2}, \overline{v}) \tag{3.84}$$

Where

$$\overline{v} = TN + \underline{v}$$

and

$$\overline{s}^2 = \frac{\sum_{i=1}^{N} (y_i - X_i^* \gamma_i)'(y_i - X_i^* \gamma_i) + \underline{v}\underline{s}^2}{\overline{v}}.$$

Nevertheless, the posterior simulator called the Gibbs sampler, involving (3.82)-(3.84), involves only random number generation from the Normal, Gamma and Wishart distributions.

The predictive densities and the marginal likelihoodsdo not existin a situation where the posterior distribution does not have a common distributional form. The normal linear model with independent normal-gamma prior becomes very complicated and difficult in Bayesian econometrics model because posterior distributional form does not have a usual form of distribution that can be solved analytically, rather via a posterior simulation approach in which the Gibbs sampler becomes an essential device of concern used in this study.

The random coefficients dynamic panel data model can be displayed in a graphical form as

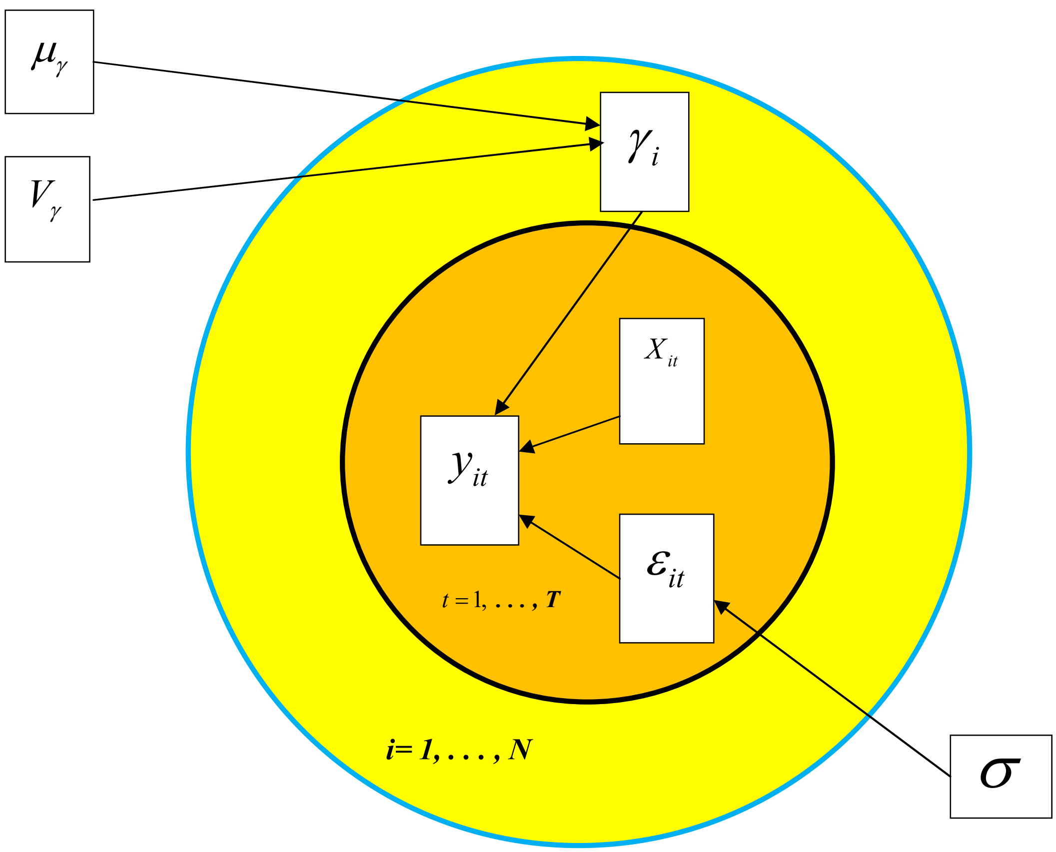

Figure 3.4: Graphical representation of the random coefficients dynamic panel data model:

$y_{it} = X_{it}^{*}\gamma_i + \varepsilon_{it}$, $i = 1, 2, \ldots, N$. Hierarchically, $\gamma_i \sim N\left(\mu_\gamma,\ V_\gamma\right)$

3.2.3 Hierarchical Bayesian Computation

3.2.3.1 Markov Chain Monte Carlo (MCMC) Approach

Markov Chain Monte Carlo (MCMC) approach is a posterior simulator used to obtain posterior estimates of the regression model of high dimensional parameter spaces. The techniques of MCMC approach to simulate posterior distributions, use Bayesian statistics especially in macroeconomic analyses and policy evaluation for estimation and prediction of empirical analysis, these two aspects of analysis are easily overcome given a sample of draws from the posterior distribution, Tanner and Wong (1987), Gelfand and Smith (1990).

The essence of MCMC approach is to incur a representative and large sample from the posterior distribution and then apply the sample to acquire information about the features of the model parameters. The sequential draws from the posterior distribution are built such that they comprise a Markov chain converging to a distribution stable which concurs with the aim posterior distribution. MCMC diagnostics are informative to measuring whether Gibbs sampler is appropriate over the study and are sufficiently of large number replicates to achieve desired degree of accuracy. The rule behind the Gibbs sampler algorithm is to partition the parameter vector into blocks of parameters and then sequentially sample from each block of parameters conditional on all the other blocks of parameters, Tierney (1994).

3.2.3.2 The Simulation Study

Simulation studies are computer experiments that include creating datarelating to numbers generated by a computational process that are from a deterministic sequence but which are designed to have as many characteristics as possible of a random sequence. The key effectiveness of simulation studies is the power to know the performance of statistical estimators because some parameters of interest are identified throughthe data generating process. This enables us to consider estimators criteria, such as numerical standard error. Although the simulation method, the setting of scenarios and the choices of the number of burn in iterations and posterior sample size are made in the process of estimating parameter model which enabled us to perform a comprehensive simulation study in a relatively short time. The focus of simulation study in this work is to examine the appropritenessof a hierarchical Bayesian estimator in the dynamic panel data in conjuction with suitable prior information.The performance of Bayesian estimation was evaluated using simulation studies of longitudinal data with different sample sizes.

3.2.3.2.1 Data Generating Scheme

The design of the Markov Chain Monte Carlo (MCMC) experiments was carried out based on the following data generating process;

Given the model,

$$y_{it} = \delta_i y_{i,t-1} + \beta_{0i} + X_{1it}\beta_{1i} + X_{2it}\beta_{2i} + \varepsilon_{it} \quad i = 1,...,N \text{ and } t = 1,...,T \text{ , k =0,1,2}$$

Markov Chain Monte Carlo (MCMC) experiments with 10,000 replications were made for the two scenarios.

To perform the simulation of the random coefficient (heterogeneous) dynamic panel data model above

(i) The explanatory variable (X_{1it}, X_{2it}) are generated using uniform distribution (0,1) that is $X_{kit} \sim U(0,1)$

(ii) The intercept is drawn independently from Normal distribution with mean zero and variance 0.25, that is $\beta_{0i} \sim N(0, 0.25)$

(iii)To impose stationarity on the coefficient lagged dependent variable (δ_i), it is assumed that δ_i are generated from Beta distribution whose support is (0, 1). Examine its stationarity condition $|\delta_i|$

(iv)The initial values for regression coefficients are $\beta_{1i} = 2$ and $\beta_{2i} = 3$

(v) The error terms are generated from normal distribution with mean zeros and one variance, that is $\varepsilon_i \sim N(0,\ h^{-1})$ of $\varepsilon_i \sim N(0,1)$. We set our error precision: *h = 25* and prior hyperparameters as $\mu_\gamma = 0_4$, $\underline{\Sigma}_\gamma = I_4$, $\underline{V}_\gamma^{-1} = I_4$ and $\underline{v}_\gamma = 2$

(vi) The first scenario has three experiments for the individual (N) and time (T):

N<T: (10, 15), (15, 20), (20, 50)

N=T: (10, 10), (20, 20), (50, 50)

N>T: (20, 5), (50, 10), (100, 15)

(vii) The second scenario improves the dimension of N>T to establish its better performance over other two experiments in first scenario as; (N, T) = (50 5), (100, 5), (100, 10), (200, 10), and (200, 20)

(viii) Markov Chain Monte Carlo (MCMC) experiments with 10,000 replicates were made for the two scenarios.

3.2.3.2.2 Data Simulation for Sensitivity of Prior Information on Posterior Estimates

To observe the behaviour of the posterior distribution, both informative and relatively non-informative priors are considered. All the prior distribution of γ_i coefficients is identical at each hierarchical prior of the model. In order to compare the effect of differently precise prior information; the error precision for the γ_i parameters priors are different between the models for the influence of the data to be examined.

(i) The independent variables (X_{1it}, X_{2it}) are generated using uniform distribution (0, 1) that is $X_{kit} \sim U(0,1)$.

(ii) The γ_i parameters: $\delta_i \sim B(0,1)$, $\beta_0 \sim N(0, 0.25)$, $\beta_{1i} = 2,\ \beta_{2i} = 3,$ are used in the simulation.

(iii) We generate values for the errors, $\varepsilon_i \sim N(0, h^{-1})$ and use the independent variables and errors to generate the dependent variable

(iv) Relatively non-informative prior hyperparameters are stated as $\underline{\gamma}_4$:

$\mu_\gamma = 0_4$, $\underline{\Sigma}_\gamma = 1$, $\underline{V}_\gamma^{-1}$=1 *, and* $\underline{v}_\gamma = 2$

(v) Informative prior hyperparameters are stated as $\underline{\gamma}_4 = \mu_\gamma = (0.5,\ 0.5,\ 0.5,\ 0.5)$, $\underline{\Sigma}_\gamma = 0.05$, $\underline{V}_\gamma^{-1} = 0.05$, *and* $\underline{v}_\gamma = 10$. To examine the prior sensitivity on the posterior distribution, we set the error precision (h=0.04, 0.03, 0.02, 0.01) for relatively noninformative prior and (h=25, 30, 50, 70) for informative prior.

(vi) The value of N (individual) is chosen to be 20 and time T=5 for the two sets of priors.

(vii) Posterior results for the model are based on 10,000 replications, with 1000 burn-in replications discarded and 9000 replications retained.

(viii) Markov Chain Monte Carlo (MCMC) experiments with 10,000 replications were made for the two scenarios.

The following criteria will be used to assess the performance of the posterior simulation techniques:

(i) Posterior Mean

The posterior simulation approach such as Gibbs Sampling provides us with $\hat{g}_{S_1}$

Where $\hat{g}_{S_1} = \frac{1}{S_1} \sum_{s=S_0+1}^{S} g(\gamma_i^{(s)})$ which is an estimate of posterior mean: $E[g(\gamma_i) \mid y]$

(ii) Posterior Standard Deviation

The posterior variance is given as:

$$\sigma_g^2 = \text{var}[g(\gamma_i, h) \mid y]$$

where posterior standard deviation can be given as:

$$\sigma_g = \left\{ \text{var}[g(\gamma_i, h) \mid y] \right\}^{1/2}$$

Geweke (1992) uses the intuition to draw on ideas from the time series literature to develop an estimate of σ_g^2 of the form:

$$\hat{\sigma}_g^2 = \frac{S(0)}{S_1}$$

(iii) Numerical Standard Error

Numerical standard error was derived through the use of a central limit theorem of the familiar form:

$$\sqrt{S_1} \left\{ \hat{g}_{S_1} - E[g(\gamma_i) \mid y] \right\} \rightarrow N(0, \sigma_g^2)$$

It is possible to calculate a numerical standard error as:

$$\frac{\hat{\sigma}_g}{\sqrt{S_1}}$$

Where

$S(0)$ is the spectral density of the sequence $\gamma_i^{(s)}$ for $s = S_0 + 1, \ldots, S$ evaluated at γ_i

S_0 is called burn- in replications which are discarded

S_1 is the remaining replicate which retained for the estimate of $E[g(\gamma_i) \mid y]$

$S_0 + S_1 = S$

(iv) Pictorial graph (Histogram and Density)

CHAPTER FOUR

ANALYSIS OF DATA AND INTERPRETATION

4.1 Introduction

In this chapter, the performance of hierarchical Bayesian estimator on dynamic panel data models when the parameters are heterogeneous across individuals are considered using the posterior mean, posterior standard deviation and numerical standard error criteria. Attention is also focussed on the two scenarios of dimension of individual (N) and time (T).

Firstly, when N<T: (10, 15), (15, 20), (20, 50)
N=T: (10, 10), (20, 20), (50, 50)
N>T: (20, 5), (50, 10), (100,15)

Secondly, improvement on performance of N>T over other two experiments in first scenario as

(N>T) = (50 5), (100, 5), (100,10), (200, 10) and (200,20)

The sensitivity of prior information on the posterior estimates of heterogeneous dynamic panel data model will also be investigated using relatively non- informative and informative priors. Data were simulated using Markov Chain Monte Carlo (MCMC) approach to obtain posterior estimates with 10,000 iterations.

4.2 First Scenario of the Empirical Analysis

The first scenario of the empirical analysis involve three experiments for the individual (N) and time (T) which are

A. N<T: (10, 15), (15,20), (20,50)
B. N=T: (10,10), (20,20), (50,50)
C. N>T: (20,5), (50,10), (100, 15)

A. Experiment I: When N<T

Table 4.1: Posterior mean and Numerical standard error (in brackets) for the second stage hierarchical prior parameters (μ_γ and V_γ);

$\mu_\gamma \mid y,\gamma,h,V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ and $V_\gamma^{-1} \mid y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$

$\delta_i \sim B(0,1)$

		N=10 , T=15	N=15 , T=20	N=20 , T=50
δ	*Mean*	-0.018960	0.126969	0.100956
	NSE	(0.00042)	(0.00037)	(0.00019)
β_0	*Mean*	-0.270528	0.220349	0.132611
	NSE	(0.00207)	(0.00142)	(0.00076)
β_1 (2.0)	*Mean*	2.761221	1.852535	1.080804
	NSE	(0.00171)	(0.00123)	(0.00017)
β_2 (3.0)	*Mean*	1.674693	1.719858	4.060692
	NSE	(0.00180)	(0.00152)	(0.0068)
V_γ		0.0033	0.0011	0.0007

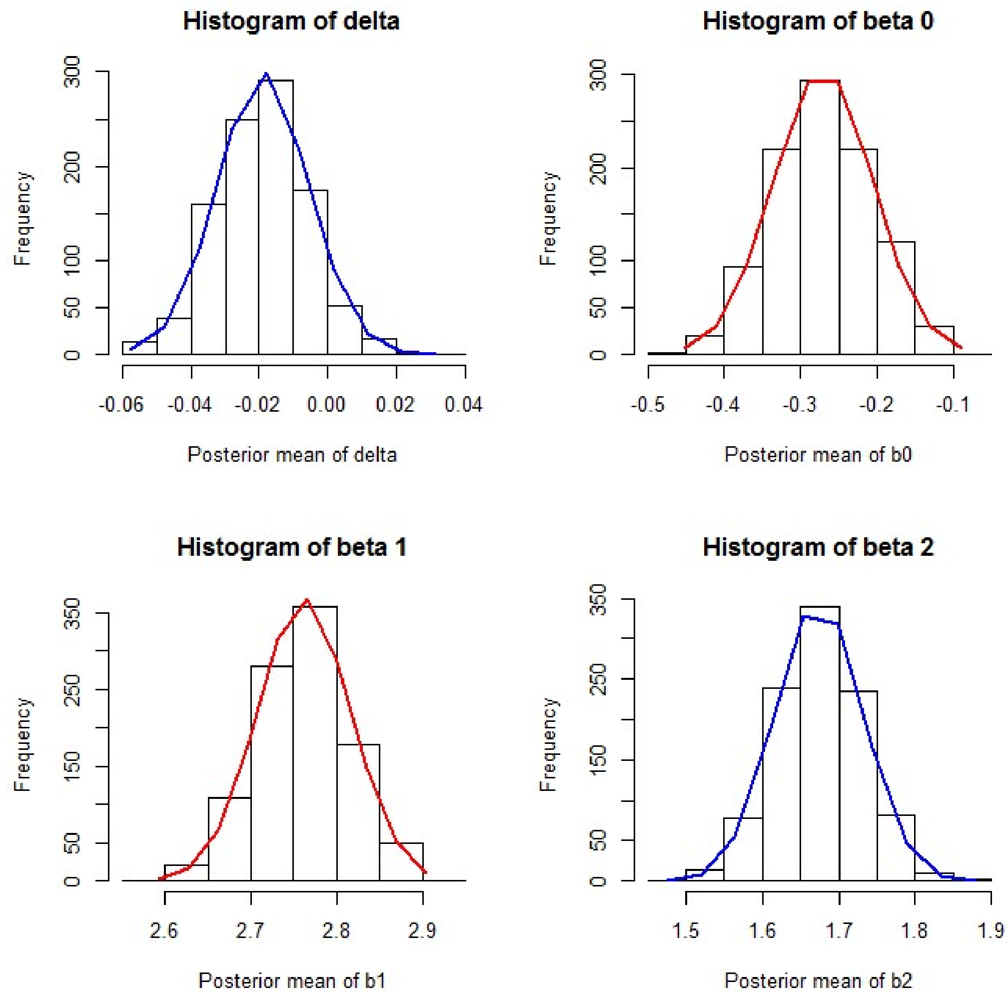

Figure 4.1(a):Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=10, T=15

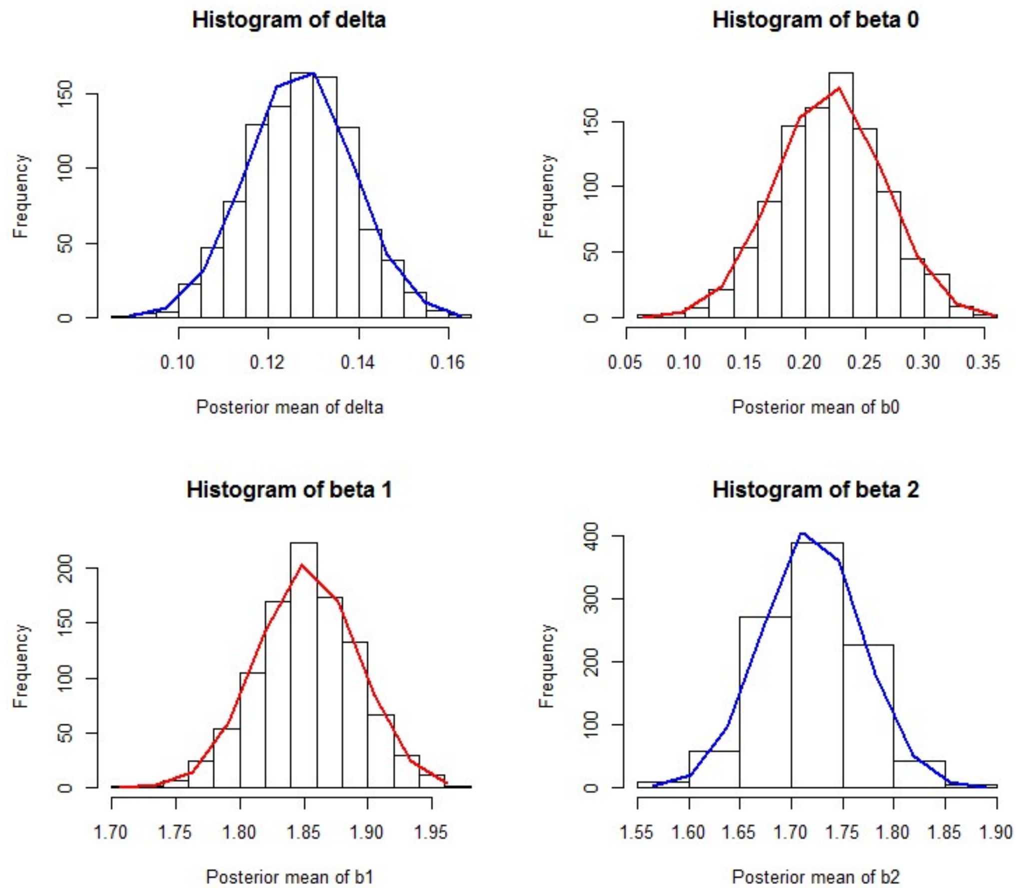

Figure 4.1(b): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=15, T=20

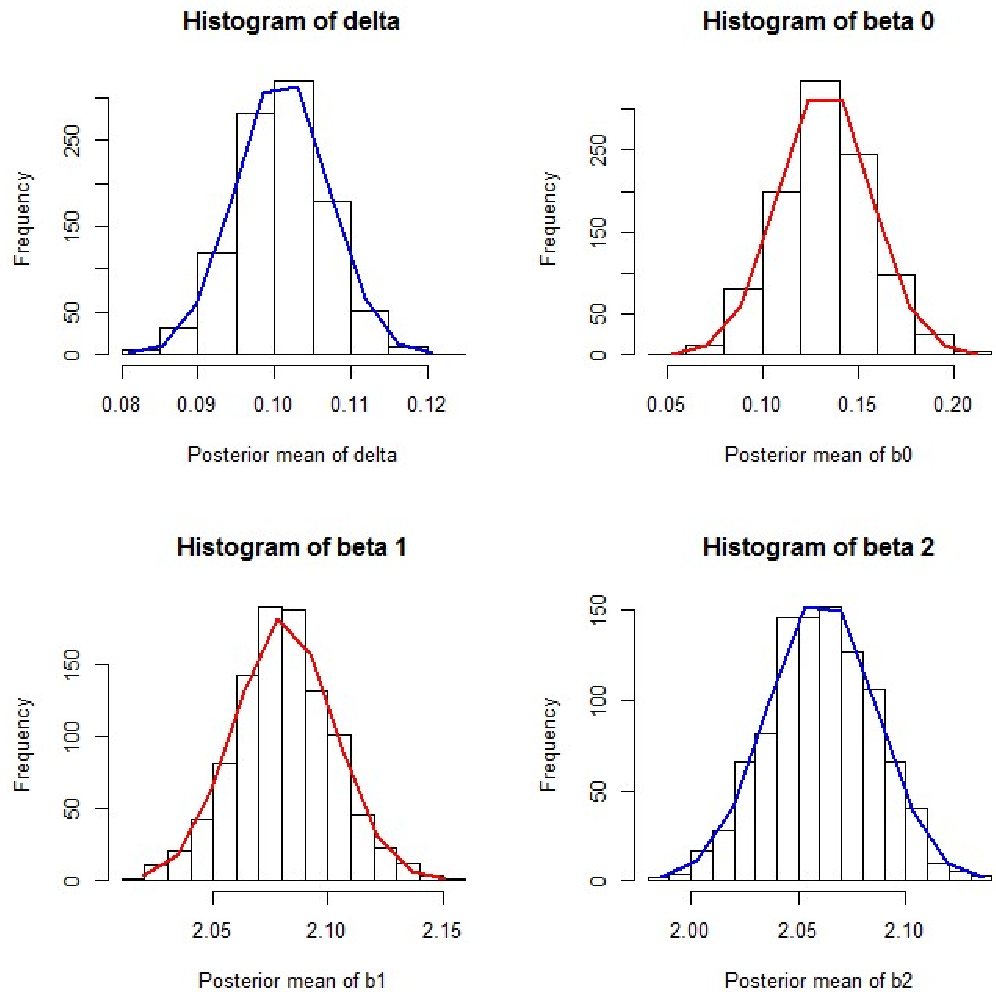

Figure 4.1.(c): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=20, T=50

Discussion of Results AI

Table 4.1 presents the posterior estimates of second stage hierarchical prior for $\mu_\gamma(\delta, \beta_0, \beta_1, \beta_2)$. It shows that the regression coefficients are identical for the entire individual (N=10, 15, 20) and time period (T= 15, 20, 50). The posterior means of the parameter δ and β_0 exhibit negative values at N=10, T=15 while the estimates of parameters β_1 and β_2 fluctuate around the true mean. The numerical standard errors of the parameters consistently decrease as N increases for all values of T. Also, the constant error variance (V_γ) which projects the homogeneity among the parameters decreases as the sample size increases.

Figures 4.1 (a – c)reveal graphical presentations of posterior estimates. The figures depict the general shapes of the marginal posterior distributions of model parameters. Histograms which superimpose a kernel density provide useful additions to the various numerical statistics for summarising MCMC output. These graphs display a marginal posterior distribution of parameters to show if the empirical distributions have normal distributions. As the dimension of N and T change, the shape of the distribution of each parameter has the normal distribution.

Table 4.2: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=10, T=15, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	-0.0198	-0.3246	2.6871	1.7403	0.0031	0.0348	0.0566	0.0838
2	-0.0197	-0.2655	2.8181	1.6056	0.0297	0.0242	0.0744	0.0509
3	-0.0297	-0.3796	2.7098	1.7918	0.0129	0.0898	0.0341	0.1353
4	-0.0272	-0.2599	2.8131	1.6440	0.0104	0.0299	0.0693	0.0125
5	-0.0219	-0.1658	2.7794	1.5998	0.0052	0.1241	0.0356	0.0567
6	-0.0046	-0.3148	2.7317	1.5776	0.0121	0.0249	0.0121	0.0789
7	-0.0381	-0.3011	2.7644	1.7434	0.0214	0.0114	0.0206	0.0869
8	-0.0007	-0.2686	2.7231	1.6182	0.0160	0.0211	0.0207	0.0384
9	-0.0175	-0.3281	2.7711	1.6381	0.0009	0.0383	0.0274	0.0184
10	-0.0120	-0.2901	2.6401	1.6063	0.0287	0.0002	0.1036	0.0502

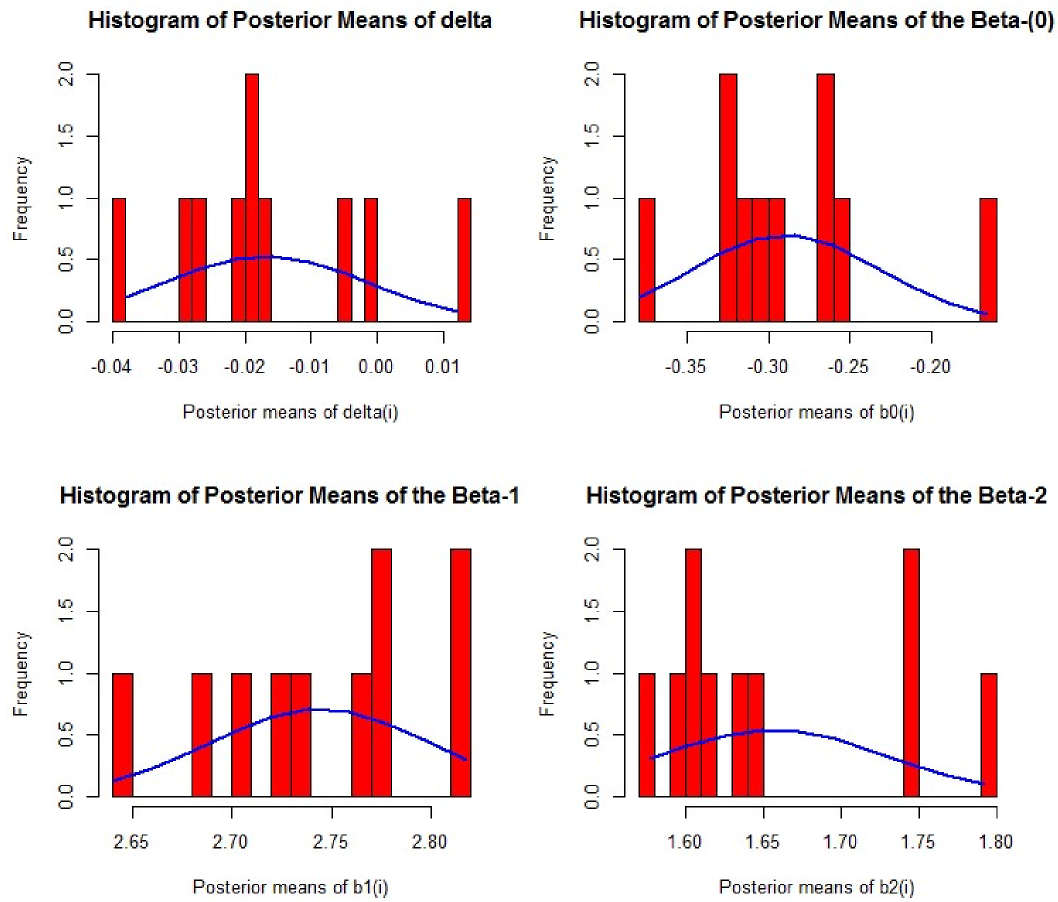

Figure 4.2: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i, \bar{V}_i)$ for N=10 and T=15

Table 4.3: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=15, T=20, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.1286	0.1538	1.8563	1.6789	0.0005	0.0570	0.0041	0.0273
2	0.1296	0.2149	1.8676	1.6749	0.0014	0.0039	0.0155	0.0313
3	0.1047	0.1724	1.8994	1.7537	0.0234	0.0384	0.0473	0.0474
4	0.1273	0.2240	1.8577	1.7270	0.0007	0.0132	0.0057	0.0207
5	0.1262	0.1914	1.8317	1.7749	0.0019	0.0195	0.0204	0.0686
6	0.1260	0.2091	1.8279	1.7247	0.0020	0.0018	0.0241	0.0183
7	0.1106	0.1732	1.9137	1.7697	0.0174	0.0377	0.0615	0.0634
8	0.1194	0.2513	1.8819	1.7081	0.0087	0.0404	0.0298	0.0018
9	0.1303	0.2396	1.8390	1.7148	0.0021	0.0288	0.0131	0.0085
10	0.1382	0.2637	1.8339	1.6988	0.0100	0.0528	0.0181	0.0074
11	0.1482	0.2006	1.7929	1.6501	0.0201	0.0103	0.0592	0.0562
12	0.1466	0.2159	1.8491	1.5985	0.0184	0.0051	0.0029	0.1078
13	0.1214	0.1951	1.8814	1.7122	0.0067	0.0157	0.0294	0.0059
14	0.1446	0.2656	1.8097	1.6325	0.0164	0.0547	0.0423	0.0737
15	0.1199	0.1923	1.8391	1.7753	0.0082	0.0185	0.0129	0.0690

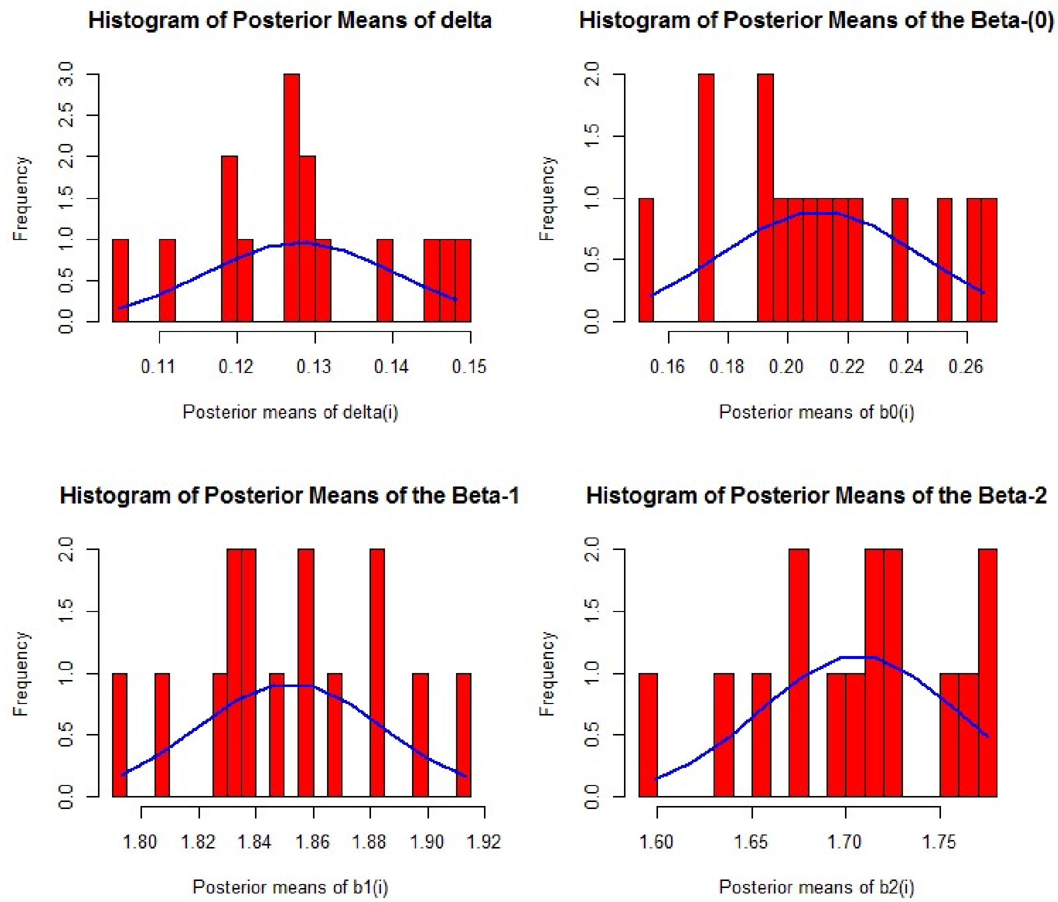

Figure 4.3: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=15 and T=20

Table 4.4: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters, $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=20, T=50, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.1023	0.1145	2.0846	2.0493	0.0015	0.0082	0.0051	0.0124
2	0.0903	0.1402	2.1292	2.0837	0.0105	0.0174	0.0497	0.0221
3	0.1000	0.1471	2.0789	2.0605	0.0007	0.0243	0.0004	0.0011
4	0.1097	0.1143	2.0795	2.0226	0.0089	0.0084	0.0002	0.0391
5	0.0981	0.1188	2.0743	2.0758	0.0027	0.0038	0.0051	0.0141
6	0.1005	0.1328	2.0832	2.0651	0.0002	0.0101	0.0037	0.0034
7	0.1063	0.1101	2.0696	2.0557	0.0055	0.0126	0.0098	0.0059
8	0.1026	0.0702	2.0718	2.0632	0.0018	0.0525	0.0076	0.0016
9	0.1023	0.1124	2.0941	2.0281	0.0015	0.0103	0.0147	0.0335
10	0.0987	0.1868	2.1005	2.0531	0.0022	0.0641	0.0210	0.0086
11	0.0929	0.0696	2.0872	2.0785	0.0079	0.0531	0.0078	0.0168
12	0.1056	0.1063	2.0480	2.0560	0.0048	0.0165	0.0313	0.0058
13	0.1036	0.1253	2.0805	2.0617	0.0028	0.0025	0.0011	0.0001
14	0.1030	0.1089	2.0762	2.0611	0.0021	0.0138	0.0031	0.0006
15	0.1033	0.1648	2.0566	2.0623	0.0024	0.0421	0.0228	0.0007
16	0.0954	0.0986	2.0512	2.1074	0.0054	0.0241	0.0821	0.0457
17	0.0983	0.1423	2.0770	2.0755	0.0025	0.0195	0.0024	0.0138
18	0.0971	0.0967	2.0946	2.0599	0.0937	0.0259	0.0152	0.0017
19	0.1003	0.1487	2.0753	2.0592	0.0005	0.0259	0.0041	0.0025
20	0.1058	0.1463	2.0586	2.0544	0.0049	0.0235	0.0035	0.0073

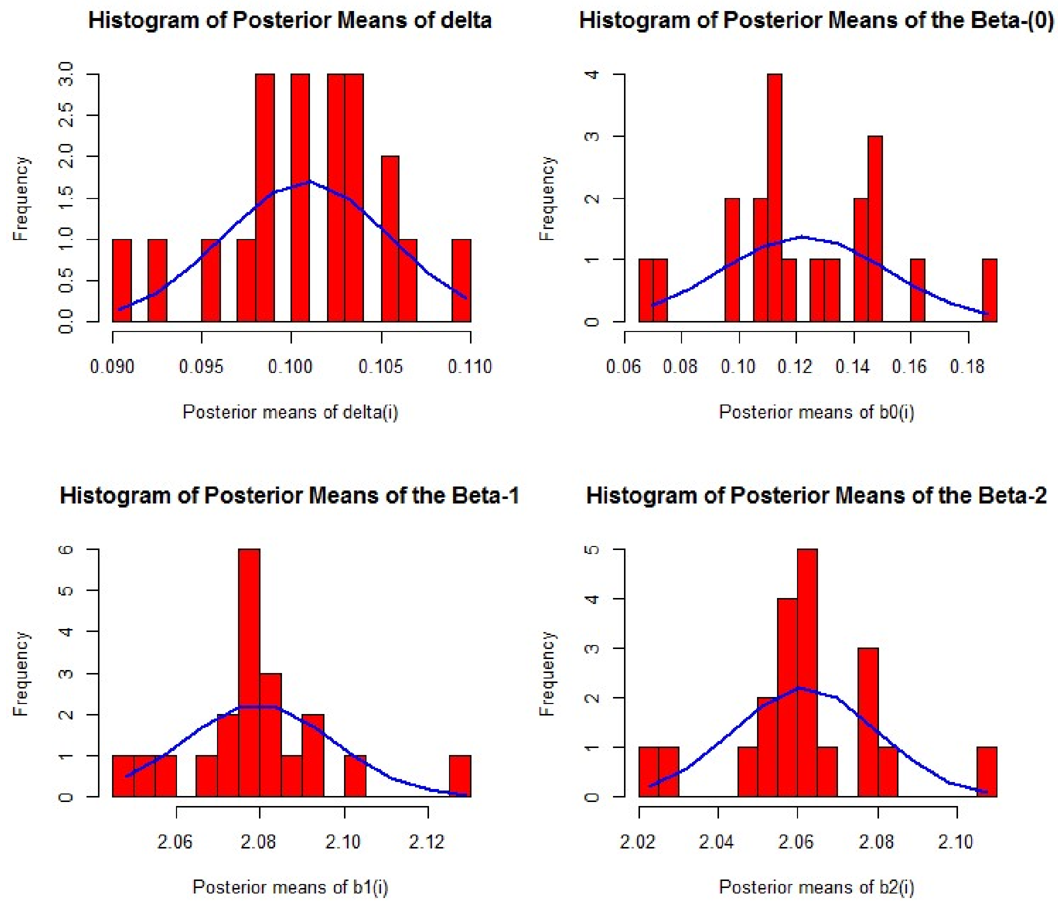

Figure 4.4: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=20 and T=50

Discussion of Results AII

Tables 4.2-4.4 present the posterior estimates of first stage hierarchical prior for $\gamma_i(\delta_i, \beta_{0i}, \beta_{1i}, \beta_{2i})$. The results obtained reveal clearly the contributions of each individual towards the response variable. Table 4.2, when N = 10 and T= 15, the individual posterior means of δ_i and β_{0i} give negative results, while posterior mean for β_{1i} is closer to the true value and the posterior mean for β_{2i} are far reaching the true mean. Table 4.3 reveals that the coefficient of lagged dependent variable (δ) possesses a stability condition, that is $|\delta|<1$ while β_{1i} and β_{2i} posterior means behave in the same manner as Tables 4.2. Table 4.4 has an improved individual posterior estimate above Table 4.2 - 4.3. More so, the posterior standard deviations for all individual parameters decrease as the sample size increases.

Figures 4.2 - 4.4 exhibit the real influence of a specific variable across the individuals. The graphs show the hidden differences among the individual parameters as their histogram is significantly different.

B. Experiment II: When N=T

Table 4.5: Posterior mean and Numerical standard error (in brackets) for the second stage hierarchical prior parameters (μ_γ and V_γ); $\mu_\gamma \mid y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ and $V_\gamma^{-1} \mid y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$, $\beta_{0i} \sim$ N(0, 0.25) $\delta_i \sim B(0,1)$

	N=10 , T=10	N=20, T=20	N=50 , T=50
δ Mean NSE	0.148184 (0.00067)	0.087397 (0.0003)	-0.015373 (0.00021)
β_0 Mean NSE	0.874782 (0.00251)	-0.213237 (0.00125)	-0.006062 (0.00052)
β_1 (2.0) Mean NSE	1.540179 (0.00229)	1.905516 (0.00108)	3.967415 (0.00014)
β_2 (3.0) Mean NSE	2.235636 (0.00268)	1.908115 (0.00127)	1.994385 (0.00048)
V_γ	0.0027	0.0007	0.0001

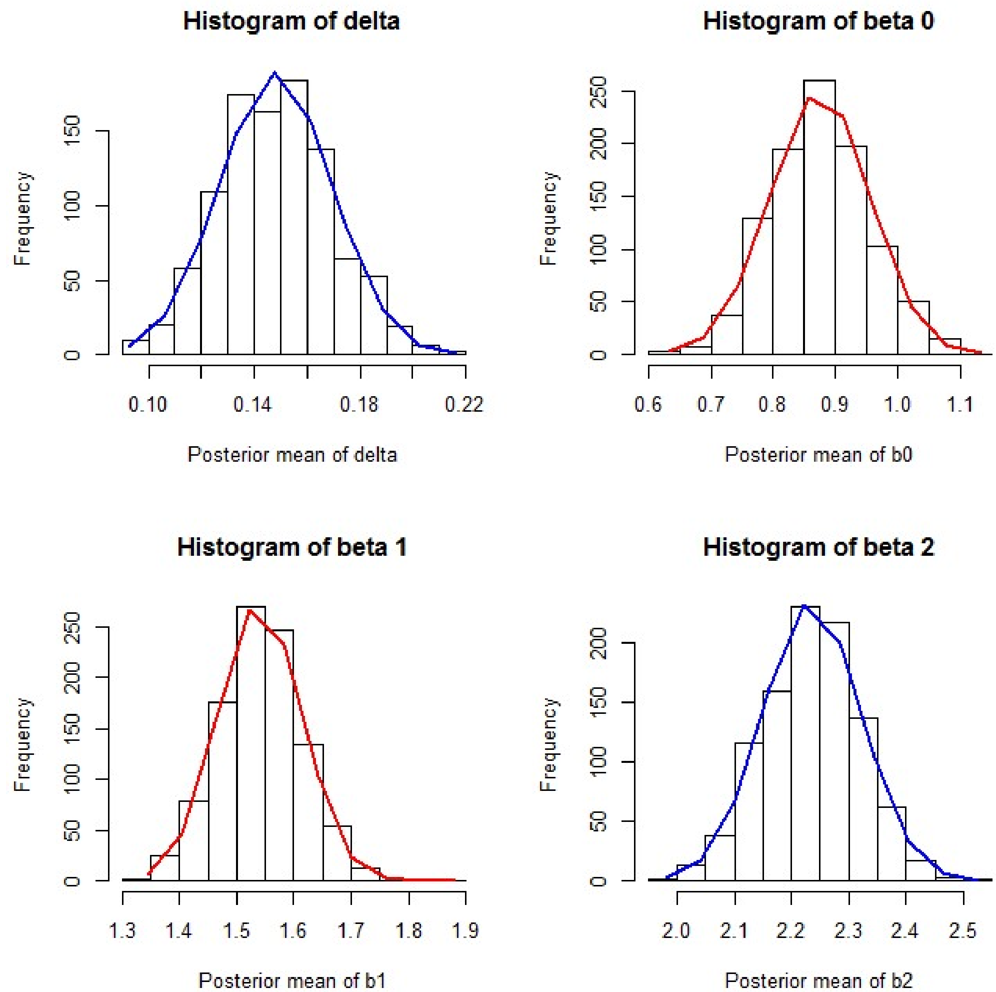

Figure 4.5(a): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=10, T=10

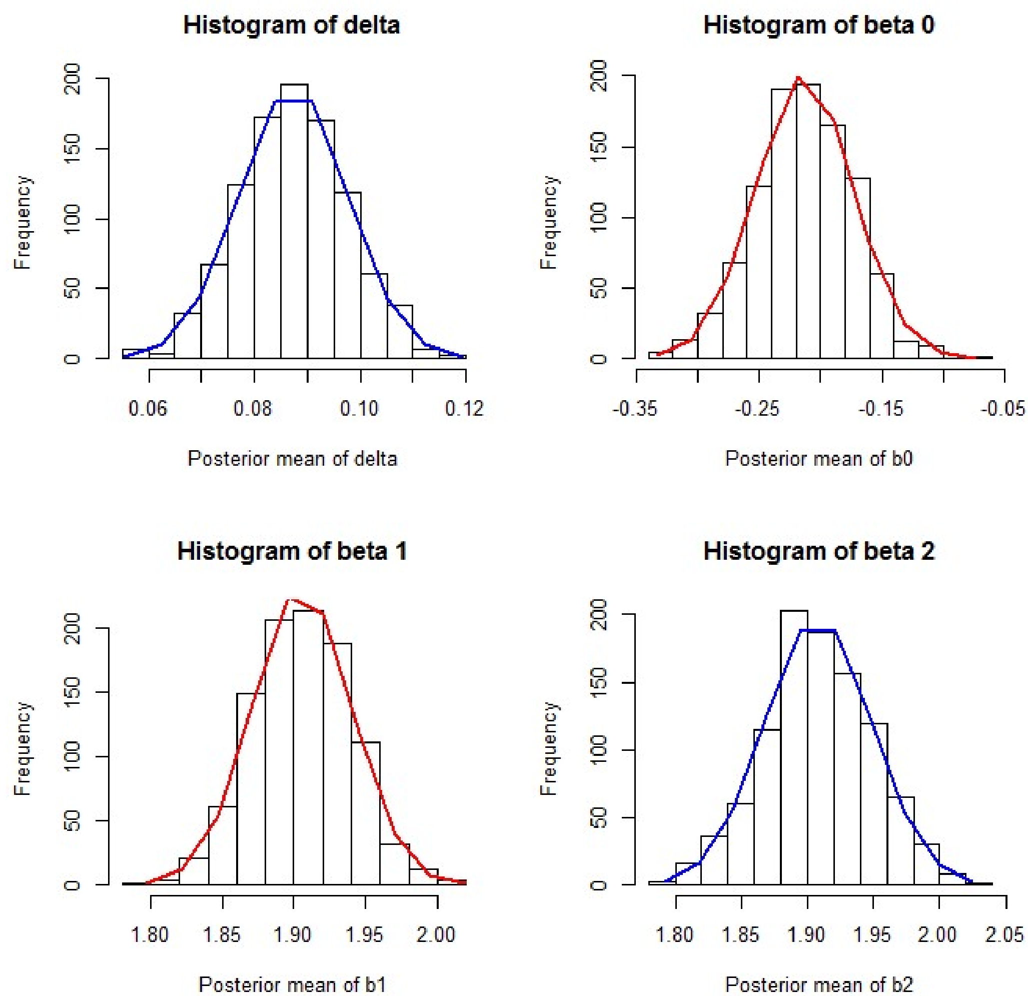

Figure 4.5 (b): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=20, T=20

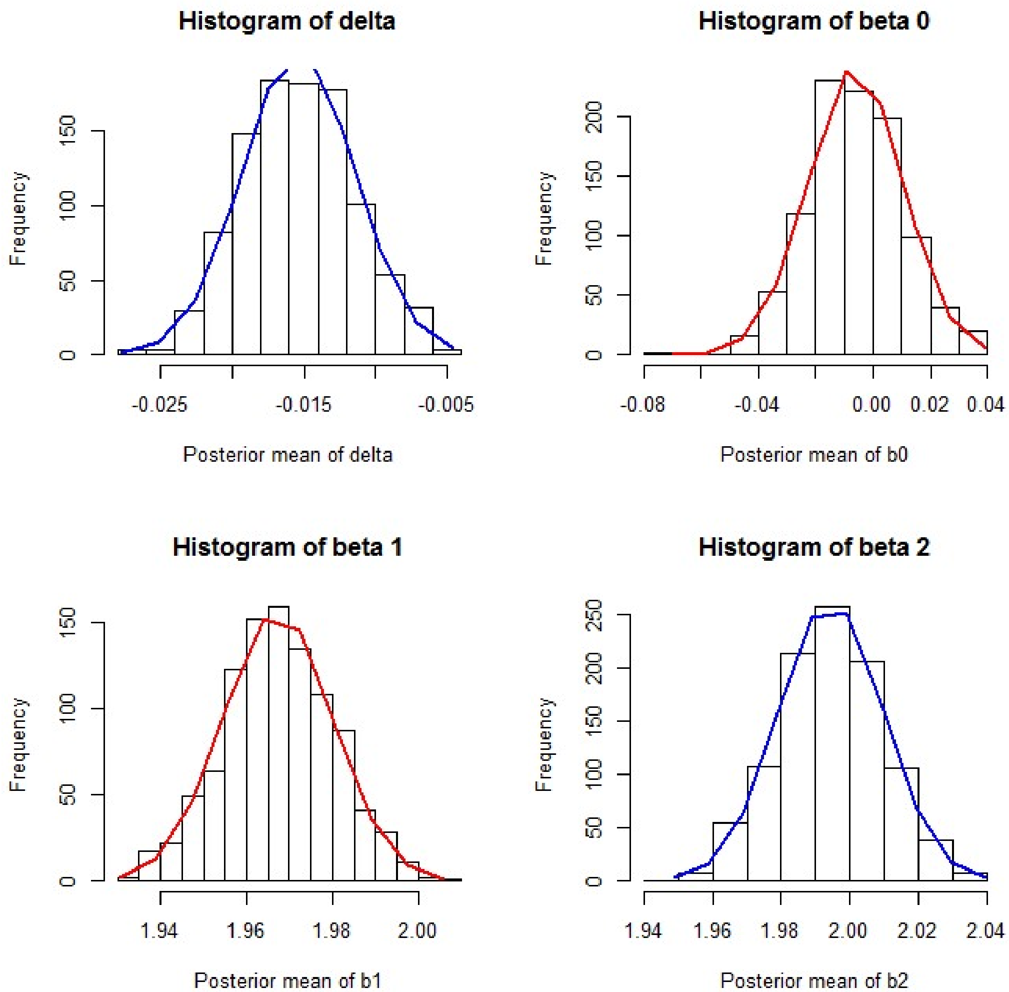

Figure 4.5 (c): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=50, T=50

Discussion of Results BI

Table 4.5 gives the posterior estimates of second stage hierarchical prior for $\mu_\gamma(\delta, \beta_0, \beta_1, \beta_2)$. The table shows that at each dimension of N=T, the regression coefficients has a single posterior mean of the entire individual (N) and time period (T). The posterior mean of the parameters δ and β_0 display negative values at N=50, T=50 while the parameters β_1 and β_2 are close to the true values at N=10, T=10 but when N=50, T=50, β_1 and β_2 are not found relatives to the true mean. The numerical standard error of each parameter decreases as every dimension of N=T increases. Also, the constant error variance (V_γ) decreases consistently as the sample size (NT) increases.

The histogram superimpose of a kernel density in Figure 4.5 (a – c) describes the pattern of each parameter posterior mean. The graph indicates marginal posterior distribution of parameters and shows that the empirical distributions have normal distributions with different shape as the dimension of N = T changes.

Table 4.6: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=10, T=10, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.1425	1.0172	1.5538	2.2193	0.0051	0.1071	0.0019	0.0117
2	0.1443	0.9123	1.5911	2.2345	0.0035	0.0022	0.0352	0.0037
3	0.1501	0.8553	1.5761	2.2004	0.0023	0.0576	0.0202	0.0307
4	0.1275	0.8414	1.5868	2.2826	0.0203	0.0685	0.0309	0.0514
5	0.1333	1.0225	1.4245	2.4231	0.0144	0.1124	0.1312	0.1919
6	0.1565	0.8764	1.5535	2.2184	0.0087	0.0335	0.0023	0.0126
7	0.1734	0.8494	1.5283	2.1866	0.0256	0.0606	0.0274	0.4455
8	0.1381	0.8553	1.6584	2.1986	0.0097	0.0548	0.1026	0.0325
9	0.1721	0.7864	1.5381	2.1254	0.0244	0.1236	0.0177	0.1057
10	0.1398	1.0843	1.5476	2.2223	0.0079	0.1742	0.0082	0.0089

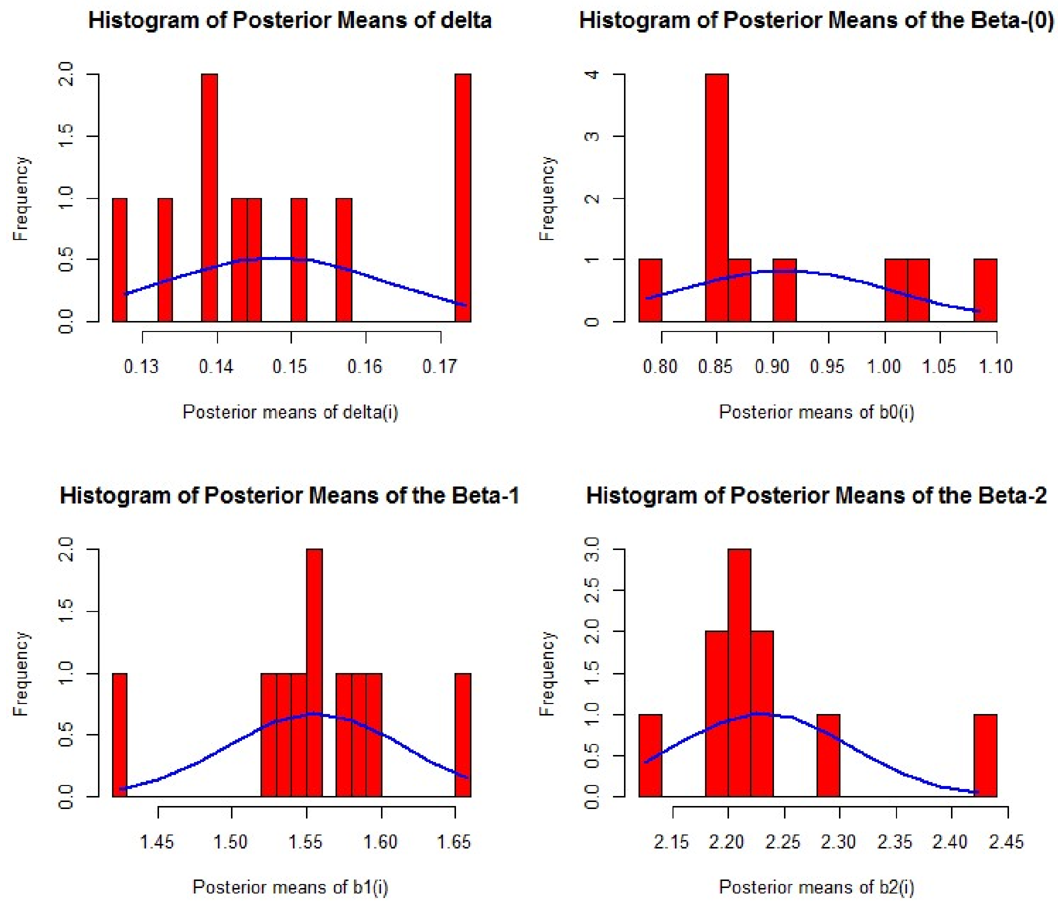

Figure 4.6: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=10 and T=10

Table 4.7: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=20, T=20, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.0919	-0.2448	1.8891	1.9069	0.0063	0.0384	0.0218	0.0015
2	0.1029	-0.2797	1.8367	1.9113	0.0172	0.0733	0.0742	0.0029
3	0.1106	-0.2361	1.8571	1.8325	0.0250	0.0297	0.0538	0.0759
4	0.0714	-0.2231	1.9498	1.9416	0.0142	0.0168	0.0389	0.0332
5	0.0945	-0.2159	1.8830	1.8708	0.0089	0.0095	0.0279	0.0376
6	0.0738	-0.2189	1.9562	1.9675	0.0119	0.0125	0.0452	0.0592
7	0.0998	-0.2461	1.8287	1.8916	0.0142	0.0398	0.0821	0.0167
8	0.0750	-0.2825	1.9324	1.9431	0.0107	0.0761	0.0214	0.0347
9	0.0803	-0.1792	1.9229	1.9340	0.0054	0.0271	0.0119	0.0256
10	0.8595	-0.2131	1.8923	1.9146	0.0003	0.0068	0.0185	0.0063
11	0.0878	-0.2085	1.9173	1.9183	0.0021	0.0022	0.0064	0.0100
12	0.0776	-0.2440	1.9586	1.9279	0.0081	0.0376	0.0476	0.0195
13	0.0685	-0.1593	1.8835	2.0242	0.0171	0.0469	0.0274	0.1159
14	0.0841	-0.1303	1.9252	1.8993	0.0016	0.0760	0.0142	0.0090
15	0.0986	-0.1821	1.9096	1.8143	0.0129	0.0242	0.0013	0.0941
16	0.0818	-0.1625	1.9292	1.9140	0.0038	0.0438	0.0183	0.0056
17	0.0765	-0.1730	1.9377	1.9519	0.0092	0.0333	0.0267	0.0435
18	0.0917	-0.1302	1.9083	1.8600	0.0060	0.0761	0.0026	0.0483
19	0.0891	-0.2140	1.9494	1.8427	0.0035	0.0077	0.0384	0.0655
20	0.0710	-0.1830	1.9514	1.9004	0.0146	0.0233	0.0405	0.0080

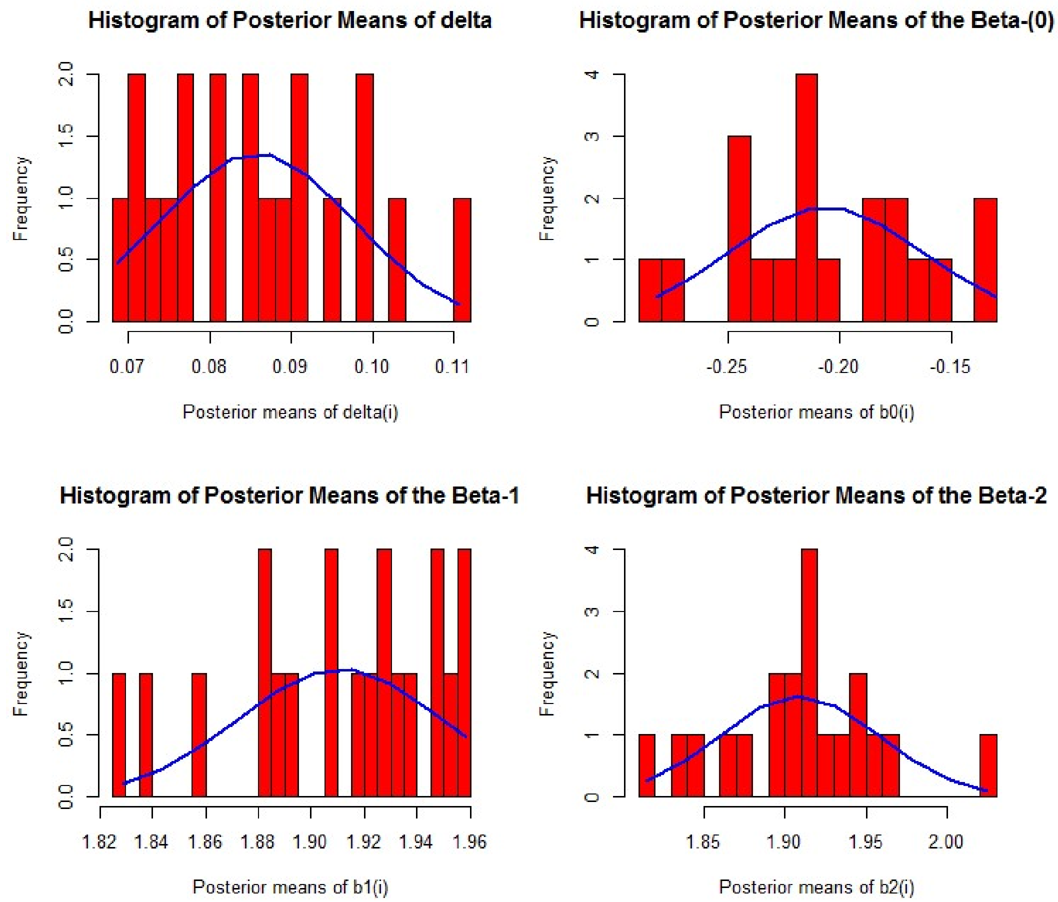

Figure 4.7: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=20 and T=20

Table 4.8: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=50, T=50, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	-0.0149	0.0405	1.9721	1.9981	0.0004	0.0431	0.0045	0.0041
2	-0.0159	0.0065	1.9507	2.0129	0.0006	0.0090	0.0168	0.0188
3	-0.0162	-0.0119	1.9493	2.0222	0.0008	0.0092	0.0183	0.0281
4	-0.0116	0.0089	1.9682	1.9759	0.0038	0.0115	0.0006	0.0182
5	-0.0114	0.0057	1.9887	1.9648	0.0039	0.0082	0.0210	0.0292
6	-0.0119	-0.0157	1.9578	1.9739	0.0034	0.0131	0.0097	0.0201
7	-0.0129	-0.0134	1.9482	2.0011	0.0024	0.0109	0.0194	0.0069
8	-0.0166	-0.0239	1.9687	1.9866	0.0013	0.0213	0.0011	0.0075
9	-0.1124	-0.0095	1.9762	1.9759	0.0041	0.0068	0.0086	0.0183
10	-0.0118	0.0120	1.9612	2.0032	0.0035	0.0146	0.0064	0.0091
11	-0.0189	-0.0262	1.9718	1.9974	0.0036	0.0237	0.0041	0.0033
12	-0.0197	-0.0291	1.9721	2.0132	0.0044	0.0265	0.0045	0.0190
13	-0.0183	-0.0086	1.9718	2.0073	0.0029	0.0061	0.0042	0.0131
14	-0.0209	-0.0055	1.9772	2.0098	0.0056	0.0029	0.0095	0.0157
15	-0.0138	0.0318	1.9577	1.9933	0.0016	0.0343	0.0098	0.0008
16	-0.0156	-0.0076	1.9597	1.9930	0.0002	0.0051	0.0079	0.0011
17	-0.0186	-0.0231	1.9653	2.0139	0.0033	0.0206	0.0022	0.0199
18	-0.0132	-0.0122	1.9666	1.9833	0.0021	0.0097	0.0009	0.0107
19	-0.0078	-0.0315	1.9517	1.9839	0.0075	0.0290	0.0159	0.0302
20	-0.0141	0.0031	1.9545	1.9882	0.0012	0.0056	0.0131	0.0059
21	-0.0152	-0.0029	1.9575	2.0041	0.0001	0.0003	0.0101	0.0099
22	-0.0129	-0.0057	1.9656	1.9949	0.0024	0.0031	0.0020	0.0008
23	-0.0169	-0.0171	1.9810	1.9989	0.0016	0.0145	0.0134	0.0047
24	-0.0161	0.0072	1.9709	1.9902	0.0008	0.0098	0.0033	0.0039
25	-0.0160	0.0165	1.9815	1.9845	0.0007	0.0187	0.0139	0.0096
26	-0.0076	-0.0281	1.9379	1.9894	0.0077	0.0256	0.0297	0.0046

27	-0.0091	-0.0058	1.9578	1.9759	0.0063	0.0032	0.0097	0.0181
28	-0.0226	-0.0079	1.9943	2.0074	0.0072	0.0053	0.0267	0.0133
29	-0.0119	0.0401	1.9771	1.9682	0.0034	0.0427	0.0094	0.0258
30	-0.0156	-0.0241	1.9797	1.9829	0.0002	0.0216	0.0121	0.0112
31	-0.0187	0.0021	1.9853	1.9905	0.0033	0.0047	0.0177	0.0036
32	-0.0171	0.0156	1.9739	2.0056	0.0017	0.0182	0.0062	0.0114
33	-0.0099	-0.0161	1.9972	1.9541	0.0055	0.0134	0.0295	0.0399
34	-0.0141	0.0016	1.9677	1.9806	0.0013	0.0042	0.0002	0.0134
35	-0.0189	0.0300	1.9760	2.0045	0.0035	0.0325	0.0084	0.0104
36	-0.0209	0.0110	1.9769	2.0031	0.0056	0.0135	0.0093	0.0090
37	-0.0151	0.0118	1.9692	1.9972	0.0003	0.0144	0.0016	0.0030
38	-0.0125	0.0157	1.9694	1.9752	0.0027	0.0183	0.0017	0.0189
39	-0.0176	0.0165	1.9500	2.0126	0.0022	0.0191	0.0175	0.0184
40	-0.0219	-0.0246	1.9778	2.0146	0.0066	0.0008	0.0101	0.0204
41	-0.0131	-0.0246	1.9588	1.9857	0.0021	0.0221	0.0088	0.0083
42	-0.0181	-0.0162	1.9491	2.0306	0.0028	0.0135	0.0185	0.0365
43	-0.0217	0.0067	1.9734	2.0265	0.0063	0.0093	0.0057	0.0323
44	-0.0169	-0.0129	1.9575	2.0077	0.0016	0.0102	0.0101	0.0135
45	-0.0092	-0.0121	1.9595	1.9714	0.0061	0.0094	0.0080	0.0227
46	-0.0187	0.0150	1.9768	1.9972	0.0034	0.0177	0.0091	0.0031
47	-0.0181	-0.0108	1.9726	1.9906	0.0028	0.0082	0.0049	0.0035
48	-0.0173	-0.0116	1.9720	1.9929	0.0019	0.0090	0.0043	0.0042
49	-0.0131	-0.0039	1.9612	1.9929	0.0022	0.0013	0.0064	0.0011
50	-0.0143	-0.0051	1.9631	2.0002	0.0011	0.0026	0.0045	0.0060

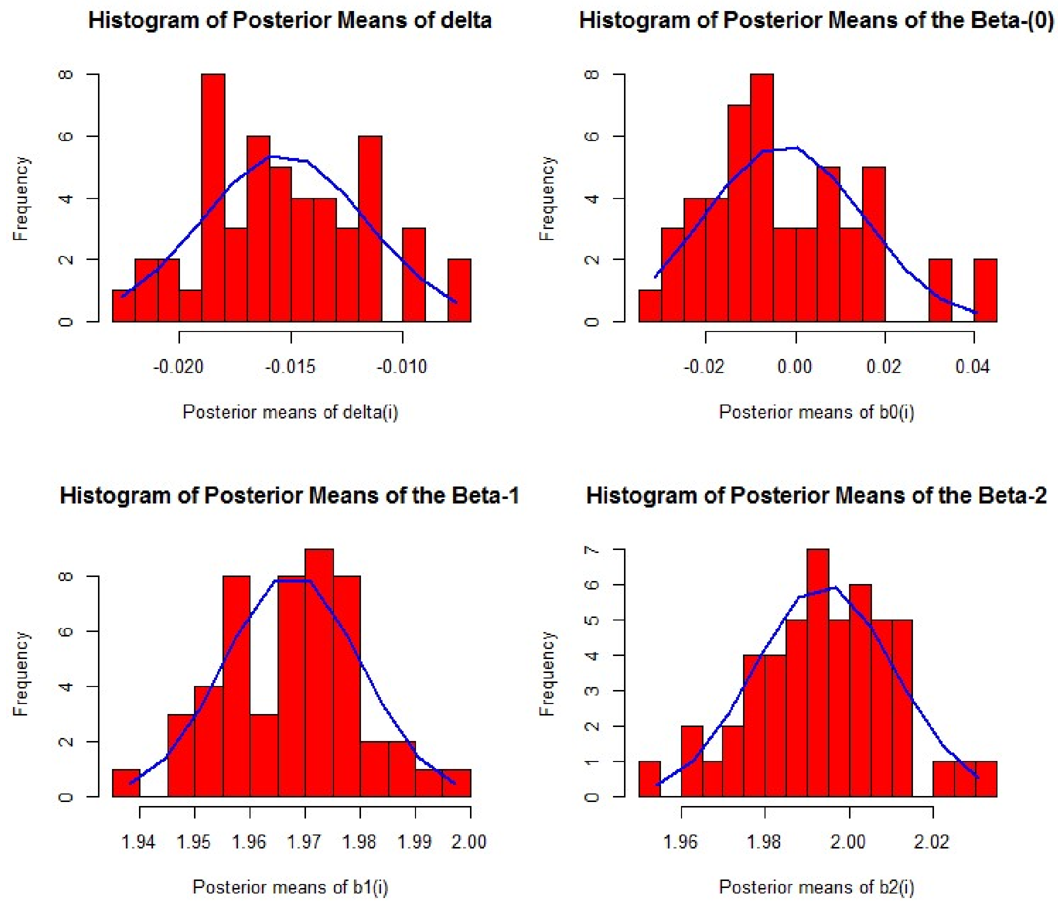

Figure 4.8: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=50 and T=50

Discussion of Results BII

Tables 4.6 - 4.8 present the posterior estimates of first stage hierarchical prior for $\gamma_i(\delta_i, \beta_{0i}, \beta_{1i}, \beta_{2i})$. The tables show the simulation results of posterior mean and posterior standard deviation of unobserved individual effects. The hierarchical Bayesian estimator perform fairly good at N=50, T=50 of the posterior mean (δ_i) for the entire individual parameters, it exhibits negative posterior mean of δ_i meanwhile, $|\delta|<1$. Also, parameters β_{1i} and β_{2i} reveal the impact of regressors across the individuals toward the dependent variable. The posterior standard deviations for every dimension of N=T considered are maximum compared to numerical standard error in Table 4.5.

Figures 4.5-4.8 display the true unobserved individual effects on dependent variable. The kernel density does not exhibit a normal distribution shape while some are flat and skewed in shapes.

C. Experiment III: When N>T

Table 4.9: Posterior mean and Numerical standard error (in brackets) for the second stage hierarchical prior parameters (μ_γ and V_γ); $\mu_\gamma \mid y,\gamma,h,V_\gamma \sim N(\overline{\mu}_\gamma,\overline{\Sigma}_\gamma)$ and $V_\gamma^{-1} \mid y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\overline{v}_\gamma,\left[\overline{v}_\gamma\overline{V}_\gamma\right]^{-1})$, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	N=20 , T=5	**N=50 , T=10**	**N=100 , T=15**
δ *Mean*	0.148197	0.0870798	0.153492
δ *NSE*	(0.00067)	(0.00028)	(0.00017)
β_0 *Mean*	0.874168	0.013789	0.163543
β_0 *NSE*	(0.00251)	(0.00109)	(0.00043)
β_1 (2.0) *Mean*	1.540353	2.016961	2.045637
β_1 (2.0) *NSE*	(0.00229)	(0.00092)	(0.00055)
β_2 (3.0) *Mean*	2.23545	2.333829	2.884716
β_2 (3.0) *NSE*	(0.00268)	(0.001195)	(0.00069)
V_γ	0.0007	0.0001	0.0000

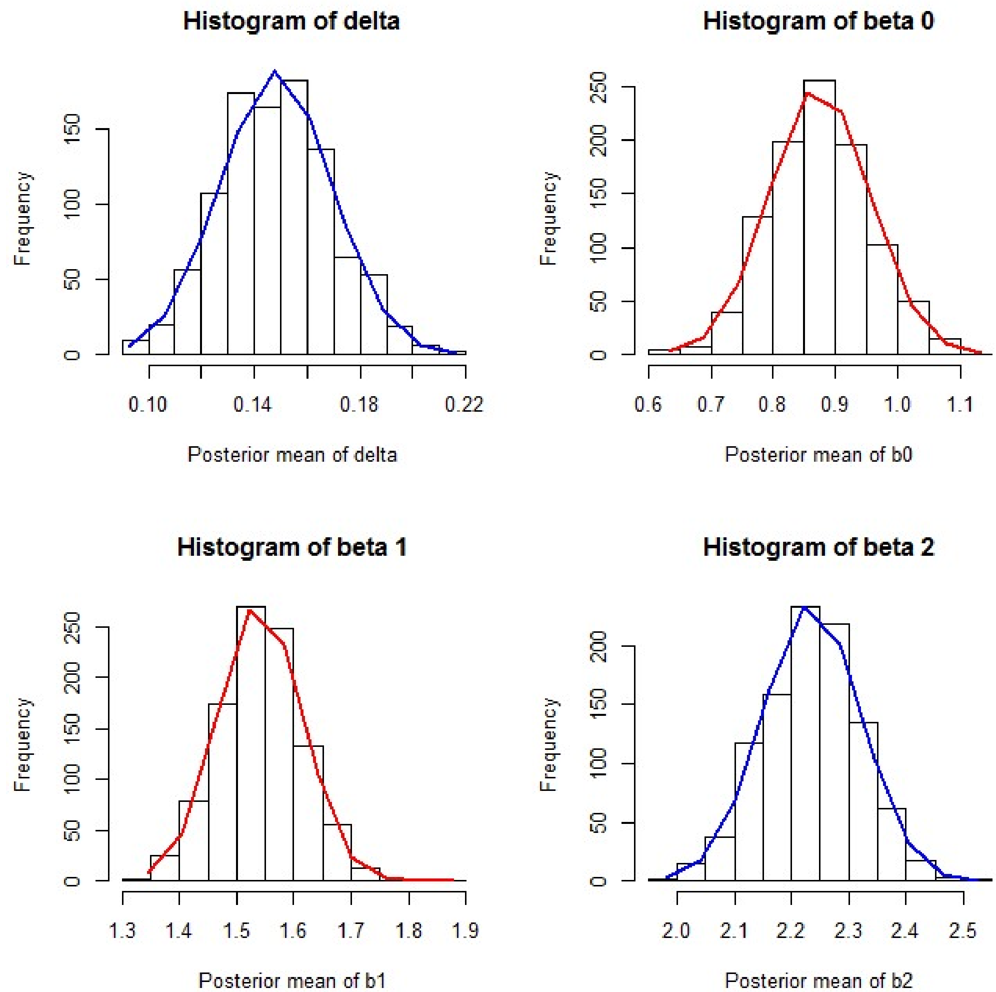

Figure 4.9 (a): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=20, T=5

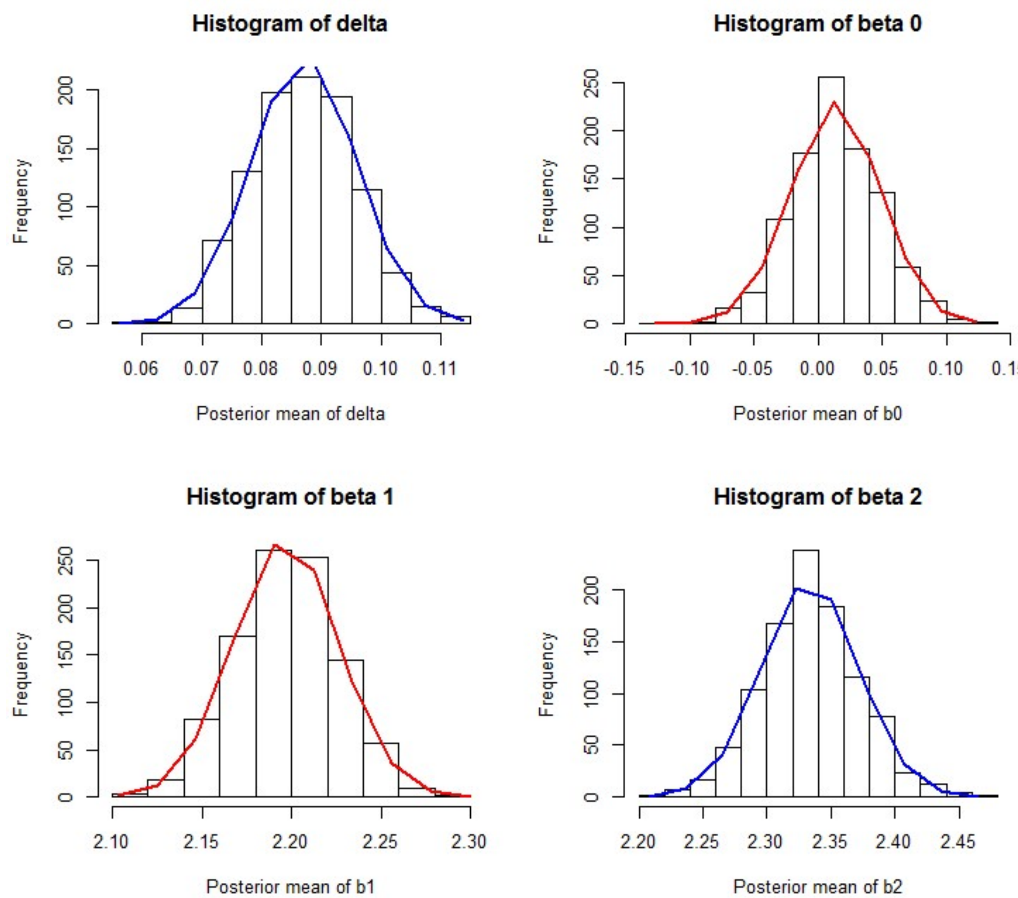

Figure 4.9(b): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=50, T=10

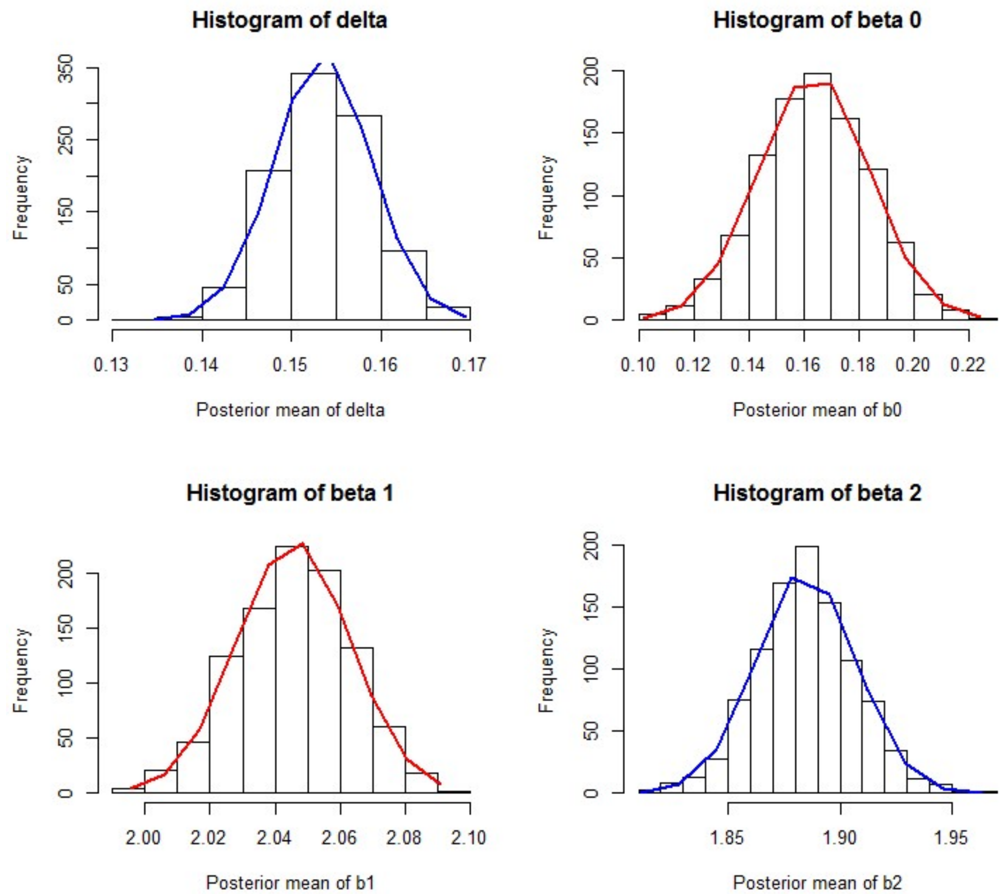

Figure 4.9(c): Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=100, T=15

Discussion of Results CI

Table 4.9 presents the posterior estimates of second stage hierarchical prior for $\mu_{\gamma}(\delta, \beta_0, \beta_1, \beta_2)$ when N>T. The table shows that the regression parameters are common for all individuals in the model (N=20, 50, 100) and time period (T= 5, 10, 15). The posterior mean of the parameter δ and β_0 are well behaved for each parameter for fall within the distribution range. The posterior mean for all parameters are consistent with the initial values of every sample size considered. The hierarchical Bayesian estimator performs better when N>T compared to N<T and N=T. None of the obtained posterior means exhibits negative results. The posterior means of β_1 and β_2 approach the true values. The numerical standard error of every parameter decreased consistently as the sample size increased. Also, the constant error variance (V_{γ}) which project the homogeneity among the parameters decreases as the sample size increases and eventually becomes zero at N=100.This indicates a good convergence of MCMC approach and shows that the error cross-sectionals are uncorrelated as $N \to \infty$.

Figures 4.9 (a – c) above reveals graphical presentations of posterior estimates. The figures for posterior mean (δ) at N=20, T=5 and β_0 at N=50, T=10 depict a peak shapes of the marginal posterior distributions. As the dimension of N and T change, the shape of the distribution of each parameter has the normal distribution of identical patterns. The figures look similar indicating that the patterns look the same for different values of N and T.

Table 4.10: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=20, T=5, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.1636	0.8460	1.4940	2.1818	0.0185	0.0267	0.0595	0.0583
2	0.1186	0.7985	1.6798	2.2720	0.0264	0.0742	0.1261	0.0318
3	0.1689	0.8326	1.5563	2.1300	0.0238	0.0401	0.0026	0.1101
4	0.1236	0.8679	1.5536	2.3523	0.0213	0.0048	0.0004	0.1121
5	0.1330	0.9214	1.5615	2.2546	0.0120	0.0486	0.0078	0.0146
6	0.1201	0.7757	1.5167	2.4114	0.0249	0.0970	0.0368	0.1712
7	0.1582	0.8579	1.5736	2.1272	0.0132	0.0148	0.0200	0.1129
8	0.1841	0.6862	1.4768	2.1038	0.0390	0.1865	0.0767	0.1363
9	0.1341	0.8867	1.5499	2.3098	0.0109	0.0139	0.0056	0.0696
10	0.1562	0.9133	1.4822	2.2337	0.0111	0.0405	0.0714	0.0064
11	0.1521	0.8523	1.5854	2.2332	0.0071	0.0204	0.0317	0.0069
12	0.1565	0.9159	1.5428	2.1937	0.0114	0.0431	0.0107	0.0464
13	0.1404	1.0213	1.5098	2.2927	0.0046	0.1485	0.0437	0.0525
14	0.1446	0.9002	1.6232	2.1767	0.0004	0.0274	0.0696	0.0634
15	0.1053	0.8419	1.6085	2.3791	0.0397	0.0308	0.0549	0.1389
16	0.1518	0.9038	1.5373	2.2435	0.0067	0.0310	0.0162	0.0033
17	0.1415	0.9180	1.6492	2.1591	0.0035	0.0452	0.0956	0.0810
18	0.1470	0.9556	1.5720	2.2305	0.0019	0.0828	0.0184	0.0096
19	0.1275	0.9837	1.4822	2.4039	0.1749	0.1109	0.0714	0.1637
20	0.1735	0.7762	1.5190	2.1142	0.0284	0.0965	0.0345	0.1259

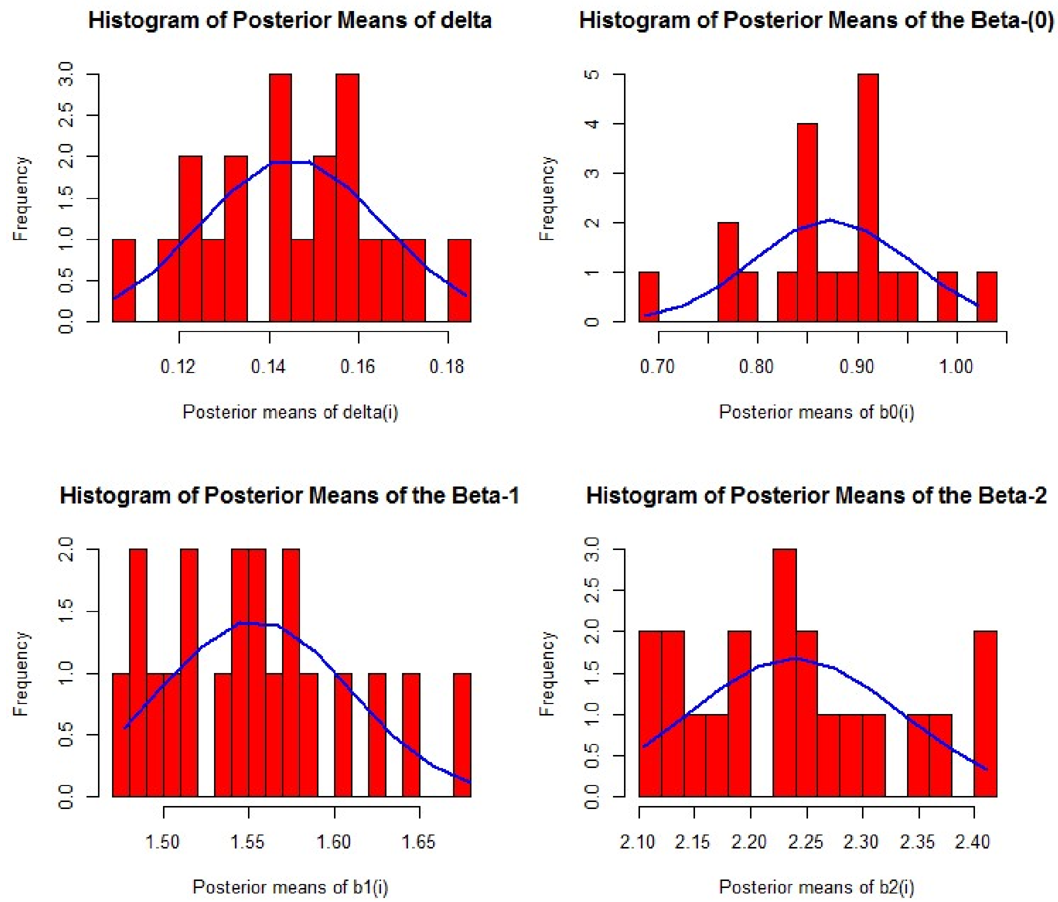

Figure 4.10: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=20 and T=5

Table 4.11: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameter $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ when N=50, T=10, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Individual	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.0866	-0.0170	2.1856	2.3581	0.0011	0.0342	0.0168	0.0191
2	0.0940	0.0640	2.1985	2.2797	0.0085	0.0468	0.0039	0.0592
3	0.0887	0.0938	2.1873	2.3039	0.0031	0.0766	0.0151	0.0350
4	0.0975	-0.0031	2.1585	2.2855	0.0120	0.0203	0.0438	0.0534
5	0.0860	0.0431	2.1927	2.3586	0.0005	0.0259	0.0097	0.0196
6	0.0741	-0.0001	2.2105	2.3884	0.0113	0.0173	0.0080	0.0494
7	0.0846	0.0046	2.1883	2.3747	0.0009	0.0125	0.0140	0.0371
8	0.0915	0.0185	2.1694	2.3379	0.0060	0.0013	0.0330	0.0010
9	0.0916	0.0285	2.1797	2.3523	0.0061	0.0113	0.0227	0.0133
10	0.0887	0.0169	2.2109	2.3280	0.0032	0.0002	0.0084	0.0109
11	0.0781	0.0430	2.1887	2.3816	0.0073	0.0258	0.0136	0.0426
12	0.0817	0.0523	2.2335	2.3312	0.0037	0.0351	0.0310	0.0077
13	0.0761	-0.0119	2.2044	2.3906	0.0093	0.0291	0.0019	0.0516
14	0.0936	0.0157	2.2202	2.2833	0.0081	0.0014	0.0177	0.0555
15	0.0819	0.0326	2.2137	2.3427	0.0035	0.0154	0.0113	0.0038
16	0.0863	0.0104	2.1659	2.3375	0.0008	0.0067	0.0365	0.0014
17	0.0847	0.0029	2.1840	2.3547	0.0007	0.0142	0.0184	0.0157
18	0.0787	0.0419	2.2540	2.3388	0.0067	0.0247	0.0515	0.0001
19	0.0877	0.0290	2.1895	2.3462	0.0022	0.0118	0.0129	0.0072
20	0.0738	-0.0294	2.2476	2.3803	0.0116	0.0466	0.0451	0.0413
21	0.0783	0.0064	2.2185	2.3699	0.0071	0.0107	0.0160	0.0309
22	0.0671	0.0058	2.2394	2.4038	0.0184	0.0113	0.0370	0.0648
23	0.0850	-0.0763	2.2311	2.2925	0.0004	0.0935	0.0286	0.0464
24	0.0788	0.0168	2.2335	2.3375	0.0066	0.0003	0.0310	0.0014
25	0.1009	0.0747	2.1599	2.2940	0.0154	0.0569	0.0425	0.0449
26	0.0962	0.0064	2.1810	2.3254	0.0107	0.0108	0.0213	0.0135

27	0.0798	0.0396	2.2022	2.3858	0.0056	0.0224	0.0002	0.0468
28	0.0861	0.0072	2.2128	2.3289	0.0006	0.0099	0.0103	0.0099
29	0.0802	0.0451	2.2073	2.3272	0.0052	0.0279	0.0049	0.0117
30	0.0809	0.0252	2.2005	2.3433	0.0046	0.0080	0.0019	0.0043
31	0.0853	0.0059	2.1971	2.3431	0.0001	0.0112	0.0053	0.0041
32	0.0713	-0.0155	2.2436	2.3710	0.0142	0.0327	0.0411	0.0320
33	0.0747	0.0334	2.2577	2.3523	0.0107	0.0162	0.0552	0.0133
34	0.0915	0.0551	2.1960	2.3123	0.0059	0.0379	0.0063	0.0266
35	0.0808	0.0284	2.2440	2.3319	0.0046	0.0112	0.0415	0.0070
36	0.0778	0.0346	2.2124	2.3545	0.0076	0.0174	0.0100	0.0155
37	0.0950	0.0421	2.2035	2.2913	0.0095	0.0249	0.0011	0.0476
38	0.0895	-0.0225	2.2216	2.3196	0.0040	0.0397	0.0191	0.0193
39	0.0856	0.0194	2.200	2.3708	0.0001	0.0022	0.0019	0.0318
40	0.1033	0.0793	2.1760	2.2707	0.0178	0.0621	0.0264	0.0682
41	0.0888	0.0443	2.2026	2.3219	0.0033	0.0271	0.0001	0.0170
42	0.0910	0.0048	2.1985	2.3239	0.0055	0.0124	0.0038	0.0150
43	0.0922	0.0171	2.1409	2.3404	0.0067	0.0000	0.0615	0.0014
44	0.0905	-0.0074	2.1878	2.3318	0.0050	0.0246	0.0145	0.0071
45	0.0774	-0.0244	2.1851	2.3747	0.0080	0.0416	0.0173	0.0357
46	0.0936	0.0264	2.2152	2.3043	0.0081	0.0092	0.0127	0.0346
47	0.0881	-0.0107	2.1915	2.3636	0.0022	0.0279	0.0109	0.0246
48	0.0904	0.0024	2.1853	2.3267	0.0049	0.0147	0.0170	0.0121
49	0.0765	-0.0348	2.1926	2.3910	0.0089	0.0520	0.0098	0.0520
50	0.0907	-0.0050	2.2002	2.2891	0.0052	0.0222	0.0021	0.0498

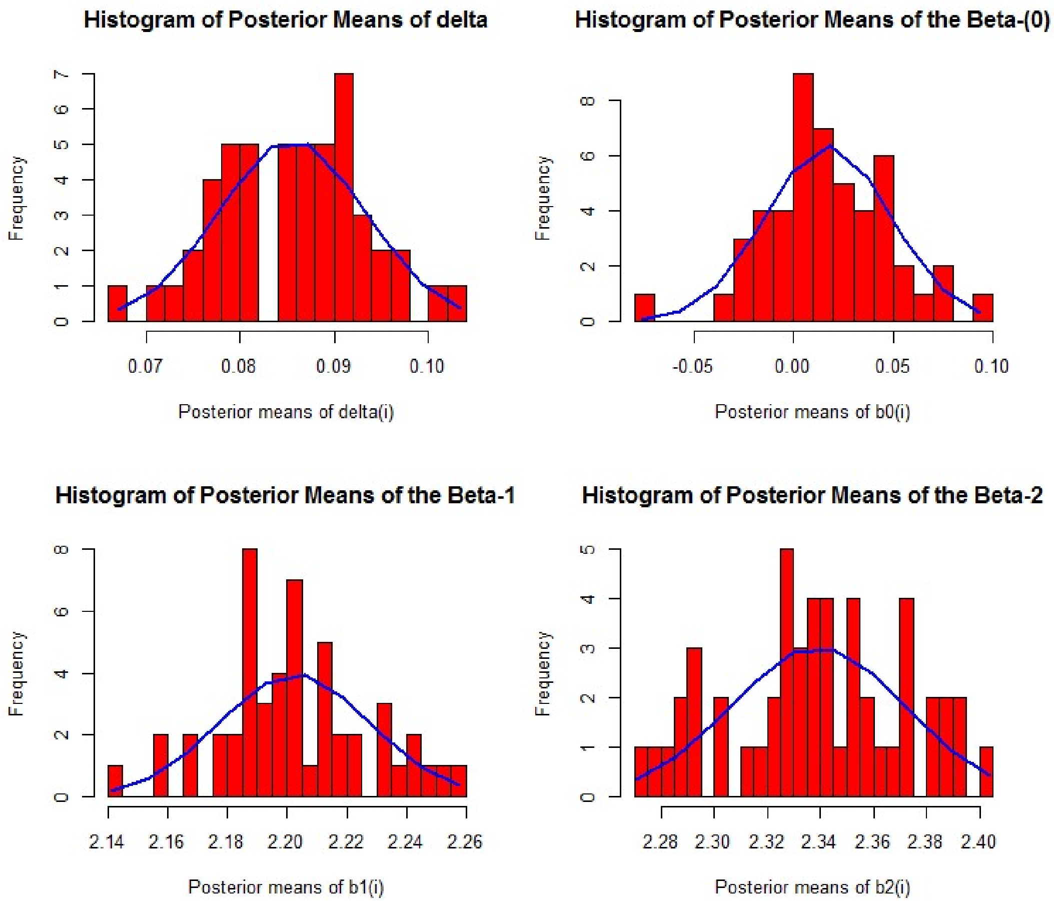

Figure 4.11: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=50 and T=10

Table 4.12: Posterior mean and Posterior Standard deviation for the first stage hierarchical prior parameters $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ For N=100, T=15, For $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior Mean				Posterior Standard deviation			
Ind	δ	β0	$\beta_1(2)$	$\beta_2(3)$	δ	β0	$\beta_1(2)$	$\beta_2(3)$
1	0.15362	0.18872	2.04768	1.88267	0.00045	0.02642	0.00073	0.00277
2	0.15097	0.14775	2.07213	1.88496	0.00220	0.01456	0.02372	0.00049
3	0.15946	0.15507	2.05688	1.84909	0.00629	0.00723	0.00847	0.03635
4	0.15875	0.21536	2.03774	1.86828	0.00558	0.05309	0.01067	0.01717
5	0.14794	0.15254	2.06859	1.90182	0.00523	0.00977	0.02018	0.01638
6	0.14678	0.22505	2.03814	1.93535	0.00639	0.06275	0.01027	0.04990
7	0.15332	0.09711	2.04999	1.88602	0.00015	0.06519	0.00159	0.00057
8	0.14923	0.16095	2.03645	1.91936	0.00394	0.00135	0.01196	0.03392
9	0.15442	0.16809	2.05282	1.88307	0.00125	0.00579	0.00441	0.00237
10	0.16241	0.13027	2.00478	1.88556	0.00925	0.03204	0.04363	0.00012
11	0.14702	0.14851	2.06981	1.89309	0.00619	0.01379	0.02140	0.00765
12	0.14896	0.15357	2.04182	1.89643	0.00421	0.00874	0.00659	0.01098
13	0.14829	0.16903	2.03806	1.93006	0.00488	0.006728	0.01035	0.04462
14	0.15394	0.17672	2.05269	1.89029	0.00077	0.01442	0.00428	0.00485
15	0.14839	0.14677	2.07135	1.90498	0.00477	0.01553	0.02294	0.01954
16	0.15733	0.14573	2.04061	1.88107	0.004165	0.01657	0.00780	0.0044
17	0.15387	0.17881	2.02430	1.91527	0.00070	0.01650	0.02411	0.02983
18	0.16199	0.20272	2.05540	1.84356	0.00882	0.04042	0.00694	0.04189
19	0.14879	0.17773	2.06405	1.89568	0.00437	0.01543	0.01564	0.01023
20	0.16316	0.20569	2.03959	1.85482	0.00992	0.04338	0.00881	0.03062
21	0.15154	0.17382	2.06851	1.87781	0.00163	0.01151	0.02009	0.0076
22	0.15346	0.15978	2.05611	1.88939	0.00029	0.00253	0.00769	0.00395
23	0.16192	0.13502	2.02164	1.84030	0.00875	0.02729	0.02677	0.04514
24	0.15570	0.18192	2.05901	1.86129	0.00253	0.01962	0.01059	0.02415
25	0.14880	0.19459	2.04605	1.90041	0.00437	0.03228	0.00236	0.01497

26	0.14092	0.16733	2.07606	1.92777	0.01225	0.00503	0.02765	0.04232
27	0.14733	0.18252	2.05181	1.90506	0.00584	0.02021	0.00339	0.01961
28	0.15569	0.16611	2.03188	1.89084	0.00252	0.00380	0.01653	0.00539
29	0.15072	0.16604	2.03914	1.90442	0.00245	0.00374	0.00928	0.01898
30	0.16238	0.14009	2.00993	1.85599	0.00921	0.02221	0.03848	0.02945
31	0.15053	0.16327	2.05982	1.87019	0.00264	0.00096	0.01141	0.01526
32	0.15957	0.15704	2.03877	1.87722	0.00639	0.00527	0.00964	0.00822
33	0.15799	0.16472	2.04336	1.87039	0.004829	0.00241	0.00506	0.01506
34	0.15507	0.15774	2.02475	1.87537	0.001903	0.00456	0.02366	0.01008
35	0.15840	0.12943	2.05493	1.84771	0.00523	0.03288	0.00652	0.03774
36	0.14713	0.16912	2.04802	1.90874	0.00604	0.00681	0.00039	0.02329
37	0.15539	0.16922	2.02848	1.89623	0.00223	0.00692	0.01993	0.01076
38	0.15089	0.14437	2.04723	1.88816	0.00228	0.01793	0.00118	0.00272
39	0.14783	0.18286	2.07025	1.90869	0.00534	0.02056	0.02185	0.02324
40	0.16046	0.15306	2.04109	1.85892	0.00729	0.00925	0.00731	0.02653
41	0.16303	0.14358	2.02809	1.85327	0.00986	0.01873	0.02032	0.03217
42	0.14735	0.19171	2.07113	1.89636	0.00582	0.02940	0.02272	0.01092
43	0.15035	0.15693	2.07428	1.88144	0.00282	0.00537	0.02587	0.00400
44	0.14589	0.19347	2.04347	1.94193	0.00728	0.03117	0.00494	0.05649
45	0.15979	0.12356	2.04101	1.85566	0.00662	0.03871	0.00740	0.02978
46	0.15493	0.11362	2.06166	1.86280	0.00176	0.04869	0.01325	0.02264
47	0.15493	0.11075	2.04939	1.97898	0.00176	0.05156	0.00099	0.00646
48	0.15353	0.12425	2.04873	1.87744	0.00037	0.03805	0.00032	0.00801
49	0.15367	0.12300	2.04204	1.88486	0.00050	0.03930	0.00637	0.00056
50	0.14613	0.15171	2.05975	1.9132	0.00704	0.01059	0.01134	0.02775
51	0.14972	0.17537	2.07028	1.87778	0.00345	0.01307	0.02187	0.00765
52	0.15508	0.16539	2.06219	1.85563	0.00191	0.00309	0.01378	0.02982
53	0.15776	0.15279	2.03541	1.85864	0.00459	0.00951	0.01299	0.02681
54	0.15146	0.116931	2.05259	1.88911	0.00171	0.04537	0.00418	0.00367
55	0.15582	0.13147	2.05182	1.87932	0.002651	0.03083	0.00341	0.00612
56	0.15128	0.20896	2.05054	1.89618	0.001891	0.04665	0.00213	0.01074
57	0.15802	0.13970	2.01205	1.89018	0.004854	0.02260	0.03636	0.00473
58	0.15015	0.15826	2.07344	1.87883	0.003024	0.00405	0.02503	0.00662

59	0.15279	0.16796	2.09096	1.87535	0.00038	0.00566	0.04255	0.01009
60	0.15753	0.15535	2.01146	1.88999	0.00436	0.00696	0.03695	0.00454
61	0.13945	0.18375	2.07304	1.93581	0.01372	0.02448	0.02463	0.05037
62	0.15524	0.16502	2.05284	1.87424	0.00204	0.00272	0.00443	0.01120
63	0.15301	0.23091	2.03968	1.89874	0.00016	0.06860	0.00873	0.01329
64	0.15427	0.16962	2.02628	1.88049	0.00109	0.00732	0.02213	0.00495
65	0.16141	0.14705	2.02606	1.86385	0.00824	0.01525	0.02235	0.02159
66	0.14596	0.17212	2.06067	1.90876	0.00728	0.00982	0.01226	0.02332
67	0.15438	0.14608	2.03629	1.88005	0.00121	0.01623	0.01212	0.00539
68	0.15383	0.19348	2.04872	1.89299	0.00067	0.03117	0.00031	0.00755
69	0.15484	0.15841	2.05800	1.85708	0.00168	0.00389	0.00959	0.02837
70	0.14219	0.18054	2.03139	1.94743	0.01098	0.01824	0.01702	0.06199
71	0.14573	0.17911	2.05897	1.91842	0.00744	0.01680	0.01056	0.03298
72	0.14267	0.15767	2.08066	1.91421	0.01050	0.00464	0.03225	0.02877
73	0.15131	0.16251	2.05212	1.88230	0.00186	0.00021	0.00371	0.00314
74	0.15191	0.18081	2.07464	1.86607	0.00125	0.01851	0.02623	0.01937
75	0.15553	0.16580	2.04453	1.88396	0.00236	0.00349	0.00388	0.00149
76	0.16127	0.15848	2.04159	1.85204	0.00810	0.00383	0.00682	0.03340
77	0.15989	0.14702	2.01099	1.87598	0.00671	0.01528	0.03742	0.00947
78	0.14812	0.16240	2.06138	1.89945	0.00505	0.00010	0.01297	0.01400
79	0.15451	0.17625	2.02871	1.87864	0.00134	0.01395	0.01970	0.00680
80	0.15147	0.15991	2.06091	1.88642	0.00169	0.00239	0.01250	0.00098
81	0.15767	0.16445	2.04204	1.86833	0.00450	0.00215	0.00639	0.01712
82	0.14869	0.18104	2.06886	1.88931	0.00448	0.01873	0.02045	0.00387
83	0.15750	0.16424	2.05099	1.86073	0.00433	0.00194	0.00258	0.02472
84	0.15333	0.18379	2.04865	1.89319	0.00016	0.02149	0.00024	0.00775
85	0.15334	0.12973	2.05015	1.88493	0.00017	0.03257	0.00174	0.00512
86	0.15564	0.16094	2.06112	1.86356	0.00247	0.00136	0.01271	0.02189
87	0.15172	0.14596	2.04209	1.88726	0.00145	0.01634	0.00632	0.00181
88	0.15091	0.16199	2.04249	1.90069	0.00226	0.00032	0.00592	0.01524
89	0.15731	0.13779	2.06385	1.85457	0.00414	0.02452	0.01544	0.03088
90	0.14146	0.16028	2.06584	1.94091	0.01171	0.00202	0.01743	0.05547

91	0.14903	0.13782	2.04081	1.89996	0.00414	0.02448	0.00760	0.01452
92	0.15993	0.17401	2.03856	1.86726	0.00677	0.01171	0.00986	0.01819
93	0.14868	0.16002	2.06548	1.88618	0.00449	0.00229	0.01707	0.00073
94	0.15473	0.15163	2.03696	1.88767	0.00157	0.01067	0.01145	0.00223
95	0.15700	0.16749	2.04203	1.86297	0.00383	0.00519	0.00638	0.02248
96	0.14751	0.14177	2.06147	1.90708	0.00565	0.02054	0.01306	0.02164
97	0.15799	0.18319	2.01941	1.87945	0.00482	0.02089	0.02900	0.00599
98	0.15861	0.18919	2.04348	1.86820	0.00544	0.02689	0.00493	0.01724
99	0.15184	0.18018	2.03704	1.89738	0.00133	0.01788	0.01138	0.01194
100	0.15746	0.14895	2.04221	1.85457	0.00429	0.01335	0.00619	0.03087

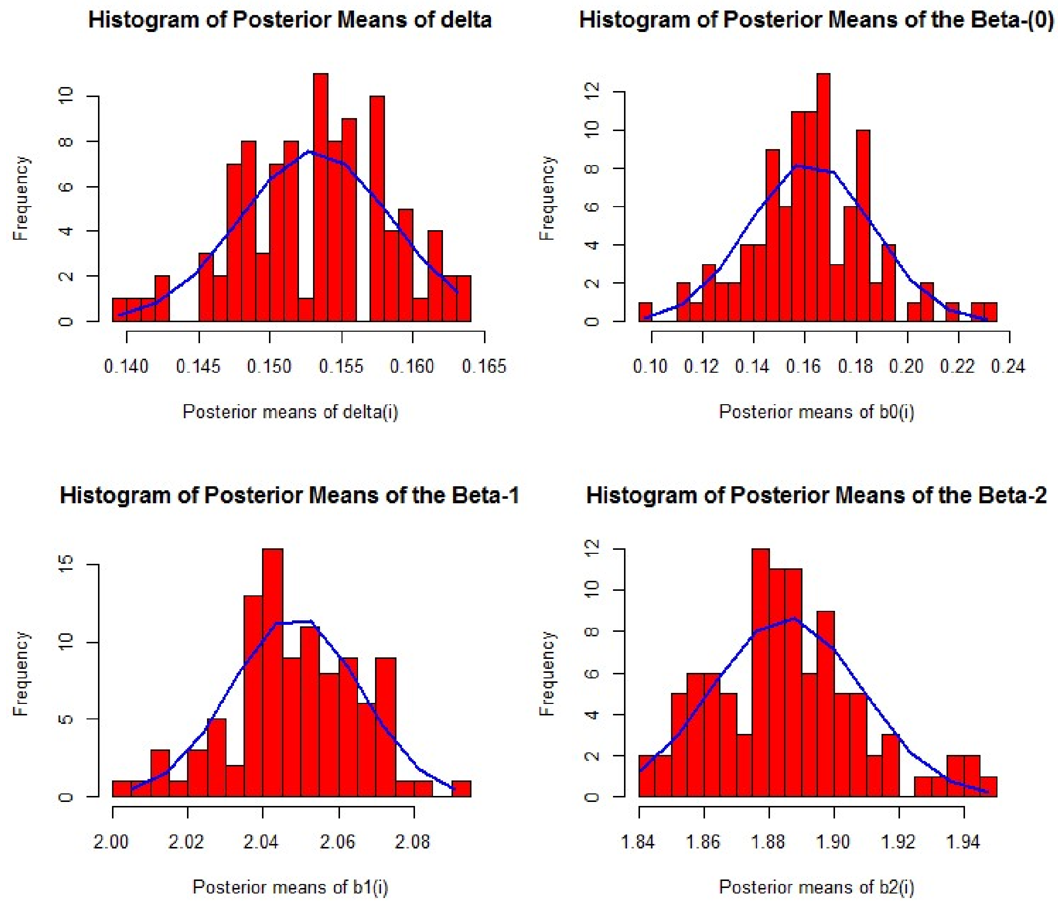

Figure 4.12: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=100 and T=15

Discussion of Results CII

Tables 4.10–4.12 present the posterior estimates of first stage of hierarchical prior $\gamma_i(\delta_i, \beta_{0i}, \beta_{1i}, \beta_{2i})$ which reveal the exact relationship that exists between dependent variables and independent variables. It is observed that the posterior means of the model parameters for all individuals exhibit the similar pattern over time period. It is worth noting that δ_i are non-negative while posterior mean of β_{1i} and β_{2i} approach their true values. The posterior standard deviations also decreased as sample size advanced across all individuals of every regression coefficients.

Figures 4.10-4.12 give very detailed information about the patterns of the model parameter at different values of N and T, the patterns exhibit a normal distribution shape compared to Figures 4.2-4.4 and Figures 4.6-4.8 as $N \to \infty$

Therefore, our findings reveal that the result of experiment III (N>T) outperforms other two experiments in terms of unbiasedness and consistency. Based on these facts, the second scenario of the study will be investigated.

4.3 Second Scenario of the Empirical Analysis

The second scenario of the empirical analysis involves dimension of the individual (N) greater than time (T); (N, T) = (50, 5), (100, 10), (200, 10), (200, 20).

4.3.1 Performance of Estimator on (N>T): (N=50, T=5)

Table 4.13: The second stage of hierarchical Bayesian Estimates: $\mu_\gamma \mid y,\gamma,h,V_\gamma \sim N(\bar{\mu}_\gamma,\bar{\Sigma}_\gamma)$ When N=50, T=5, h=25, $\beta_{0i} \sim$ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.07218707	0.02074556	1.91806187	2.10788077	17.47170448
Standard deviation	0.01261984	0.04887839	0.04089667	0.05229691	2.43211366
Numerical Standard Error	0.000399075	0.00154567	0.00129326	0.001653774	0.076910187

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} \mid y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\bar{v}_\gamma,\left[\bar{v}_\gamma \bar{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.02141 & -0.00003 & -0.00002 & 0.00004 \\ -0.00003 & 0.02135 & 0.00004 & 0.00017 \\ -0.00002 & 0.00004 & 0.02155 & 0.00004 \\ 0.00004 & 0.00017 & 0.00004 & 0.02134 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00441 & 0.00325 & 0.00326 & 0.00307 \\ 0.00325 & 0.00462 & 0.00319 & 0.00327 \\ 0.00326 & 0.00319 & 0.00457 & 0.00314 \\ 0.00307 & 0.00327 & 0.00314 & 0.00461 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00014 & 0.00010 & 0.00010 & 0.00009 \\ 0.00010 & 0.00015 & 0.00010 & 0.00010 \\ 0.00010 & 0.00010 & 0.00015 & 0.00009 \\ 0.00009 & 0.00010 & 0.00009 & 0.00015 \end{bmatrix}$$

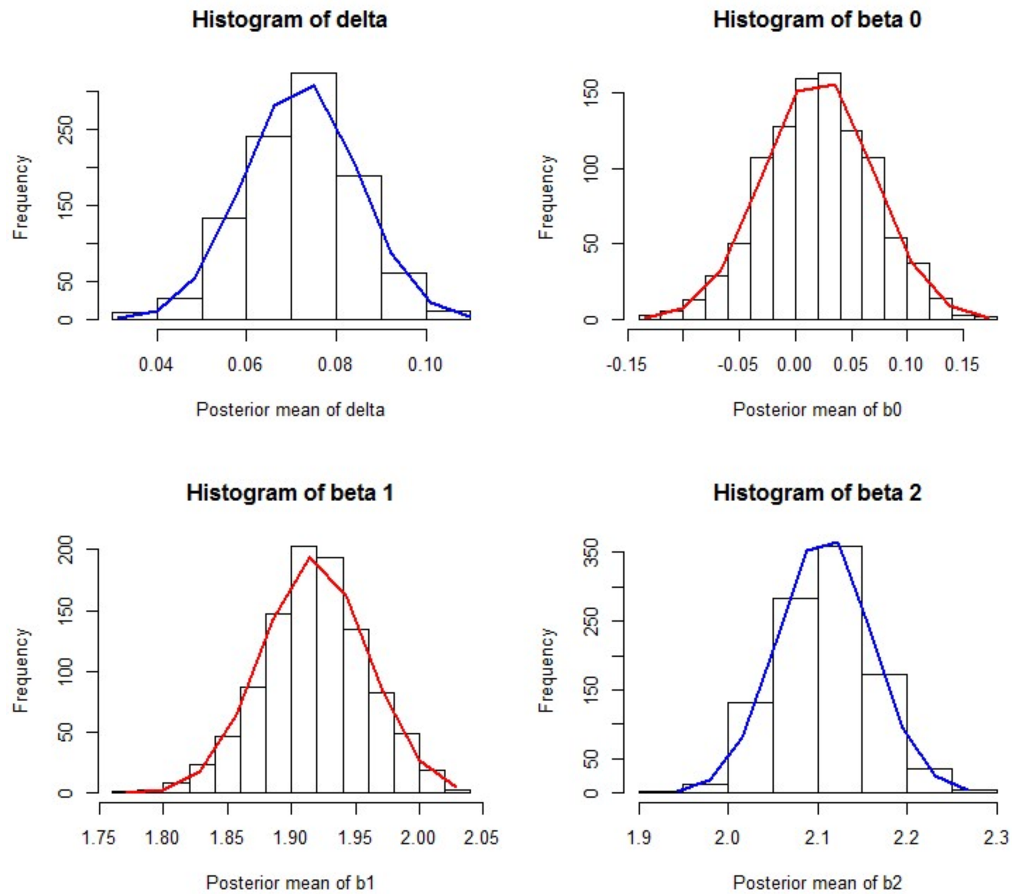

Figure 4.13: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=50, T=5

Table 4.14: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=50, T=5, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Ind.	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.07358	-0.02565	1.83795	2.14661	0.05571	0.05997	0.09265	0.03121
2	0.06444	0.08625	1.94537	2.11449	0.00343	0.05193	0.01476	0.00091
3	0.06260	0.11452	1.93332	2.16197	0.00526	0.08020	0.00259	0.04658
4	0.05142	0.07941	1.99516	2.12224	0.01644	0.04509	0.06455	0.00684
5	0.07241	0.05235	1.98161	2.04238	0.00454	0.01803	0.05100	0.07301
6	0.04602	0.07090	1.93496	2.19153	0.02184	0.03658	0.00436	0.07614
7	0.05603	0.04069	1.96837	2.13468	0.01183	0.00637	0.03776	0.01928
8	0.06178	-0.00453	1.93557	2.12401	0.00608	0.03885	0.00496	0.00861
9	0.08661	-0.00954	1.88705	2.01874	0.01874	0.04386	0.04355	0.02866
10	0.04559	0.07211	1.96371	2.16776	0.02227	0.03779	0.03310	0.05237
11	0.05459	0.07220	1.94618	2.20138	0.01327	0.03788	0.01557	0.08598
12	0.05765	0.09510	1.94645	2.15023	0.01021	0.06079	0.01586	0.03483
13	0.07064	0.03162	1.90173	2.16609	0.00276	0.00269	0.02887	0.05070
14	0.06770	0.03687	1.95442	2.04331	0.00016	0.00255	0.02381	0.07208
15	0.04752	0.02535	1.97016	2.18496	0.02034	0.00897	0.01773	0.06956
16	0.04845	0.03664	1.94834	2.19839	0.00455	0.07894	0.01635	0.08300
17	0.06685	0.07908	1.89044	2.17847	0.00066	0.03682	0.01636	0.06308
18	0.07242	-0.04462	1.94759	2.05981	0.00156	0.00020	0.02911	0.05558
19	0.06721	-0.00251	1.91424	2.14406	0.00467	0.01887	0.02561	0.02866
20	0.06630	0.03411	1.95972	2.05818	0.00179	0.00648	0.02024	0.05720
21	0.07253	0.01544	1.90499	2.15087	0.00221	0.00132	0.00644	0.03547
22	0.06966	0.02783	1.91036	2.14318	0.00112	0.00263	0.00453	0.02779
23	0.08150	-0.03112	1.96748	2.01519	0.01363	0.06544	0.03687	0.10019
24	0.06359	0.03319	1.93774	2.11973	0.00427	0.00112	0.00714	0.00440
25	0.07353	0.05172	1.87455	2.13596	0.00566	0.01739	0.05605	0.02057
26	0.09365	-0.01784	1.90267	1.99469	0.02589	0.05216	0.02793	0.12069

27	0.06480	0.08751	1.96601	2.10029	0.00307	0.05320	0.03541	0.01510
28	0.06575	0.06366	1.92353	2.14116	0.00211	0.02934	0.00707	0.02577
29	0.07251	0.08876	1.88710	2.11079	0.00464	0.05444	0.04359	0.00459
30	0.04273	0.01542	1.97719	2.24856	0.02514	0.01889	0.04659	0.13316
31	0.09354	-0.01915	1.88835	1.99927	0.02566	0.05347	0.04224	0.11611
32	0.07869	-0.09493	1.93139	2.08159	0.01082	0.12924	0.00079	0.03380
33	0.07032	0.01297	1.93188	2.08937	0.00245	0.02134	0.00127	0.02601
34	0.08070	0.05432	1.91427	2.04554	0.01283	0.02001	0.01633	0.06985
35	0.09661	0.06819	1.87028	2.03895	0.02874	0.03387	0.06032	0.07643
36	0.07825	0.00958	1.89285	2.08654	0.01039	0.02473	0.03775	0.02885
37	0.06802	-0.00108	1.89942	2.16728	0.00016	0.03539	0.03118	0.05188
38	0.03824	0.00437	1.99790	2.17705	0.02962	0.02994	0.06729	0.06165
39	0.04396	0.08524	2.00966	2.22819	0.02389	0.05092	0.07902	0.11281
40	0.08749	0.00319	1.83984	2.12089	0.01962	0.03112	0.09076	0.00550
41	0.06901	0.13043	1.94630	2.10783	0.00114	0.09612	0.01569	0.00757
42	0.07297	0.02433	1.91539	2.09219	0.00510	0.00998	0.01521	0.02320
43	0.05370	0.11503	1.97580	2.12752	0.01416	0.08070	0.04519	0.01213
44	0.05766	0.06420	2.01867	2.10431	0.01559	0.05305	0.01286	0.01108
45	0.08346	0.08736	1.91775	2.05729	0.00452	0.01264	0.00134	0.05810
46	0.07239	0.04696	1.92926	2.08129	0.01572	0.04464	0.04390	0.03409
47	0.05214	-0.01032	1.97450	2.17493	0.00906	0.06492	0.05519	0.05954
48	0.07693	-0.03061	1.87541	1.87541	0.04248	0.02972	0.03432	0.01000
49	0.08035	0.01060	1.89924	2.03630	0.01246	0.02371	0.03136	0.07909
50	0.09877	-0.01981	1.88826	1.99004	0.03089	0.05413	0.04234	0.12535

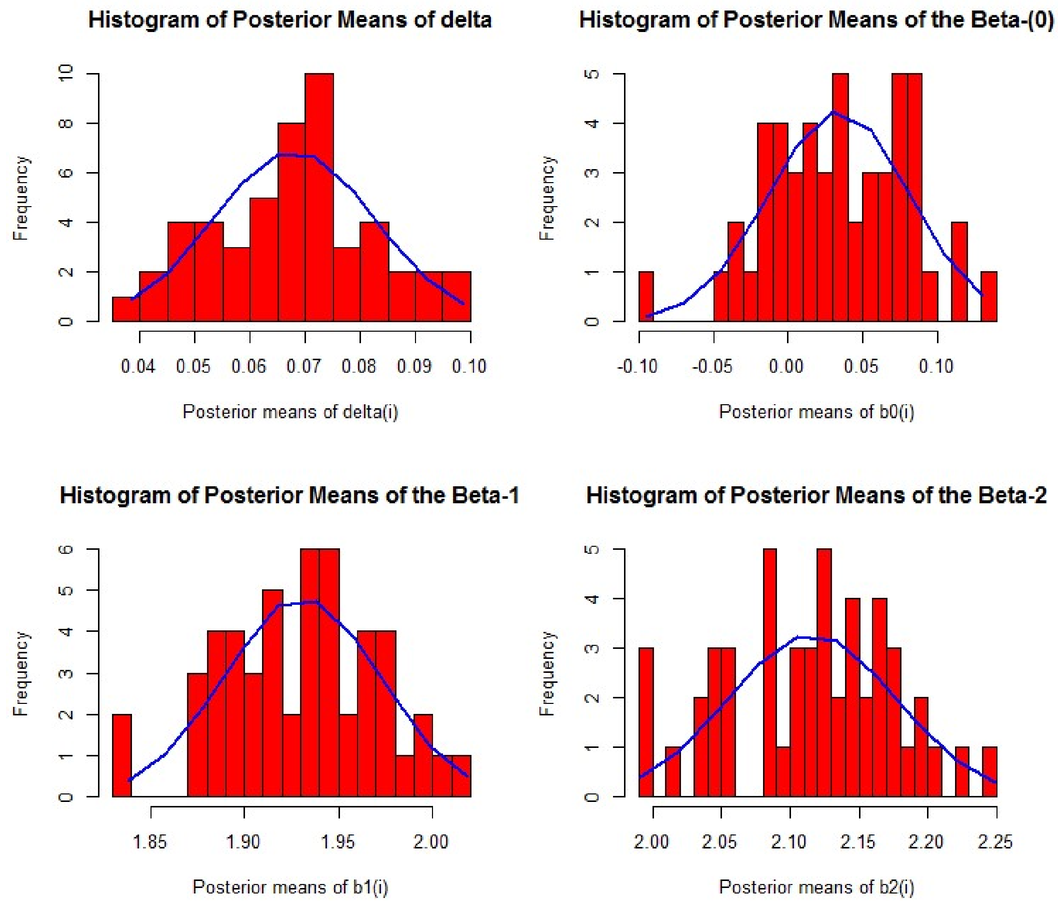

Figure 4.14: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=50 and T=5

Discussion of Results (Tables 4.13 & 4.14 and Figures 4.13 & 4.14)

Table 4.13 presents results for the second stage hierarchical prior. The table reveals the posterior estimates that regression coefficients are common for entire individual (N). The obtained values for β_{1i} and β_{2i} show good estimates as it was set of the initial values 2 and 3. The posterior estimates of variance-covariance matrix for V_{γ} are presented and noticed that numerical standard error are smaller to standard deviation of all the parameters.

Table 4.14 gives posterior estimates of all the parameters across the individual as each approach the true value. Figure 4.13 presents the histogram graph of each parameter of the posterior means, while Figure 4.14 displays a normal distribution shape which picks out the variation in the regression coefficients. Hence perfect constant error variance was not recorded for all the parameters in the model.

4.3.2 Performance of Estimator on (N>T): (N=100, T=10)

Table 4.15 The second stage of hierarchical Bayesian Estimation:
$\mu_\gamma \mid y,\gamma,h,V_\gamma \sim N(\bar{\mu}_\gamma,\bar{\Sigma}_\gamma)$ When N=100, T=10, h=25, β_{0i} ~ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.1009522	0.1325840	2.0809505	2.0605782	24.566376
Standard deviation	0.0059320	0.0239983	0.021264790	0.02531231	2.28856164
Numerical Standard Error	0.000187589	0.000758895	0.0006724517	0.00080044	0.13561643

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} \mid y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\bar{v}_\gamma,\left[\bar{v}_\gamma\bar{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.01034 & 0.00003 & -0.00006 & -0.00002 \\ 0.00003 & 0.01028 & -0.00000 & -0.00002 \\ -0.00006 & -0.00000 & 0.01025 & -0.00001 \\ -0.00002 & -0.00002 & -0.00001 & 0.01025 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00142 & 0.00106 & 0.00106 & 0.00099 \\ 0.00106 & 0.00150 & 0.00098 & 0.00106 \\ 0.00106 & 0.00098 & 0.00150 & 0.00102 \\ 0.00099 & 0.00106 & 0.00102 & 0.00150 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00005 & 0.00003 & 0.00003 & 0.00003 \\ 0.00003 & 0.00005 & 0.00003 & 0.00003 \\ 0.00003 & 0.00003 & 0.00005 & 0.00003 \\ 0.00003 & 0.00003 & 0.00003 & 0.00005 \end{bmatrix}$$

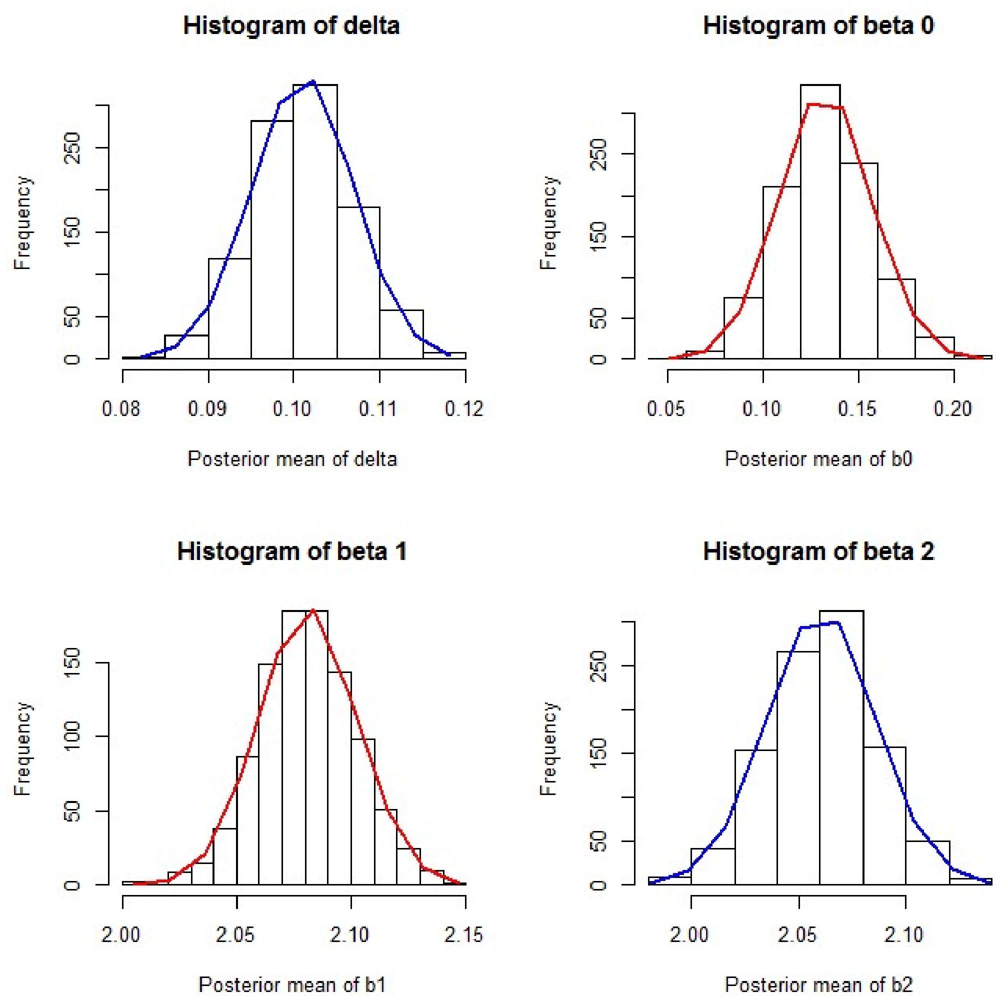

Figure 4.15: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=100, T=10

Table 4.16: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=100, T=10, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior Mean				Posterior Standard deviation			
Individual	δ	β0	$\beta_1(2)$	$\beta_2(3)$	δ	β0	$\beta_1(2)$	$\beta_2(3)$
1	0.10559	0.13397	2.05168	2.05857	0.00482	0.00777	0.03021	0.00356
2	0.10715	0.13164	2.10159	2.02486	0.00838	0.00544	0.01970	0.03726
3	0.10393	0.10362	2.08728	2.04670	0.00316	0.02258	0.00539	0.01542
4	0.10533	0.14628	2.07259	2.05323	0.45585	0.02008	0.00929	0.00889
5	0.09743	0.12253	2.12908	2.03539	0.00339	0.00367	0.04721	0.02673
6	0.09607	0.17476	2.12470	2.06334	0.00469	0.04856	0.04282	0.00122
7	0.10100	0.14282	2.07993	2.06799	0.00023	0.01662	0.00196	0.00587
8	0.09580	0.14416	2.09597	2.05694	0.00497	0.01796	0.01408	0.00518
9	0.09324	0.08886	2.12364	2.05652	0.00753	0.03734	0.04175	0.00561
10	0.11077	0.12841	2.07097	2.01039	0.01000	0.00221	0.01092	0.05174
11	0.10835	0.11729	2.05842	2.04120	0.00758	0.00891	0.02347	0.02092
12	0.12142	0.15614	2.02699	1.99002	0.02065	0.02993	0.05490	0.07211
13	0.10257	0.08789	2.08513	2.06435	0.00181	0.03832	0.00324	0.00222
14	0.09793	0.09768	2.08448	2.06797	0.00284	0.02852	0.00259	0.00585
15	0.10103	0.08182	2.05097	2.09814	0.00026	0.04437	0.03092	0.03616
16	0.10894	0.13621	2.06822	2.01531	0.00817	0.01000	0.01367	0.04681
17	0.10155	0.12656	2.07365	2.04297	0.00079	0.00035	0.00827	0.01916
18	0.09706	0.09133	2.12525	2.05348	0.00371	0.03487	0.04336	0.00864
19	0.10468	0.15002	2.06097	2.05429	0.00392	0.02382	0.02092	0.00784
20	0.09854	0.15204	2.07891	2.06470	0.00223	0.02584	0.00299	0.00258
21	0.10219	0.13310	2.08890	2.05244	0.00142	0.00689	0.00702	0.00969
22	0.09007	0.11179	2.10086	2.08631	0.01069	0.01442	0.01897	0.02419
23	0.09409	0.13086	2.09003	2.09787	0.00667	0.00466	0.00814	0.03575
24	0.09795	0.12777	2.11416	2.03001	0.00282	0.00157	0.03227	0.03211
25	0.10916	0.06799	2.07196	2.03069	0.00839	0.05821	0.00993	0.03144

26	0.10241	0.12267	2.07927	2.06079	0.00164	0.00354	0.00262	0.00133
27	0.11094	0.13534	2.06386	2.00451	0.01018	0.00914	0.01803	0.05761
28	0.09698	0.14512	2.08164	2.05632	0.00378	0.01892	0.00025	0.00579
29	0.09493	0.15980	2.09067	2.09056	0.00583	0.03359	0.00878	0.02843
30	0.10107	0.13493	2.05452	2.09263	0.00030	0.00873	0.02737	0.03051
31	0.08792	0.13087	2.09015	2.10136	0.01285	0.00467	0.00826	0.03935
32	0.10351	0.11559	2.06362	2.06685	0.00274	0.01061	0.01827	0.00473
33	0.10259	0.10549	2.07559	2.06184	0.00206	0.02071	0.00629	0.00028
34	0.09600	0.16949	2.09076	2.10166	0.00475	0.04329	0.00887	0.03954
35	0.10100	0.15182	2.08634	2.07309	0.00023	0.02562	0.00445	0.01097
36	0.10266	0.12526	2.08119	2.05482	0.00189	0.00095	0.00069	0.00731
37	0.09614	0.15425	2.09336	2.06297	0.00463	0.02804	0.01147	0.00085
38	0.09575	0.11094	2.09112	2.08826	0.00502	0.01526	0.00931	0.02613
39	0.10305	0.14327	2.08549	2.04423	0.00228	0.01706	0.00359	0.01789
40	0.10443	0.09845	2.07099	2.04069	0.00366	0.02775	0.01089	0.02143
41	0.10001	0.13321	2.08793	2.06047	0.00075	0.00701	0.00604	0.00165
42	0.11013	0.14089	2.05341	2.05416	0.00936	0.00147	0.02848	0.00796
43	0.09502	0.14454	2.09508	2.07110	0.00487	0.01833	0.01318	0.00898
44	0.09676	0.12532	2.07787	2.09550	0.00401	0.00088	0.00402	0.03338
45	0.09429	0.10593	2.10260	2.07209	0.00648	0.02027	0.02071	0.00997
46	0.09899	0.10909	2.08014	2.05906	0.00177	0.01712	0.00175	0.00307
47	0.10275	0.11859	2.09962	2.04998	0.00199	0.00761	0.01773	0.01215
48	0.10417	0.13384	2.05756	2.06757	0.00340	0.00764	0.02433	0.00545
49	0.10932	0.09989	2.06315	2.02452	0.00855	0.02631	0.01874	0.03761
50	0.10492	0.07953	2.10176	2.03373	0.00415	0.04667	0.01987	0.02839
51	0.10254	0.11215	2.09141	2.04655	0.00177	0.01405	0.00952	0.01557
52	0.10085	0.13413	2.06229	2.06845	0.00008	0.00792	0.01959	0.00632
53	0.11034	0.10039	2.05977	2.02715	0.00958	0.02580	0.02212	0.03498
54	0.09967	0.13674	2.08164	2.08309	0.00109	0.01054	0.00025	0.02097
55	0.10811	0.13519	2.06919	2.05670	0.00734	0.00899	0.01270	0.00542
56	0.09710	0.09708	2.10577	2.05657	0.00367	0.02912	0.02388	0.00556
57	0.09249	0.13188	2.07305	2.11832	0.00828	0.00567	0.00884	0.05619
58	0.09756	0.08753	2.09386	2.06470	0.00321	0.03868	0.01197	0.00258

59	0.10403	0.09741	2.06325	2.07651	0.00326	0.02879	0.01864	0.01439
60	0.09950	0.13146	2.08170	2.08889	0.00127	0.00526	0.00019	0.02677
61	0.10207	0.15789	2.07704	2.05852	0.00131	0.03169	0.00485	0.00361
62	0.08821	0.12890	2.11619	2.08895	0.01256	0.00270	0.03430	0.02683
63	0.10479	0.13643	2.04683	2.04433	0.00402	0.01022	0.03506	0.01779
64	0.09441	0.18428	2.05939	2.11491	0.00628	0.05808	0.02249	0.05278
65	0.09103	0.14738	2.09143	2.08910	0.00973	0.02118	0.00954	0.02697
66	0.10421	0.12623	2.06092	2.05855	0.00344	0.00002	0.02097	0.00358
67	0.10331	0.11295	2.05535	2.07758	0.00255	0.01325	0.02654	0.01545
68	0.09618	0.14727	2.12892	2.07048	0.00459	0.02107	0.04703	0.00836
69	0.10234	0.16699	2.10277	2.03184	0.00158	0.04079	0.02088	0.03028
70	0.09635	0.13485	2.06919	2.09502	0.00442	0.00865	0.01269	0.03289
71	0.09237	0.17945	2.07953	2.11000	0.00839	0.05324	0.00236	0.04788
72	0.10635	0.14278	2.08965	2.03914	0.00558	0.01658	0.00776	0.02298
73	0.10442	0.10789	2.07966	2.02925	0.00366	0.0183	0.00223	0.03288
74	0.09757	0.13628	2.10614	2.03871	0.00320	0.01008	0.02425	0.02341
75	0.09639	0.11261	2.09592	2.08251	0.00347	0.01359	0.01403	0.02039
76	0.09926	0.12096	2.08011	2.07129	0.00150	0.00523	0.00178	0.00917
77	0.09574	0.16454	2.11619	2.06398	0.00502	0.03834	0.03431	0.00186
78	0.10279	0.08713	2.09093	2.05005	0.00203	0.03907	0.00904	0.01207
79	0.10284	0.14324	2.06294	2.06028	0.00208	0.01738	0.01895	0.00184
80	0.09828	0.11268	2.08772	2.06179	0.00249	0.01357	0.00583	0.00033
81	0.09612	0.10083	2.08368	2.09006	0.00464	0.02537	0.00179	0.02793
82	0.10141	0.07683	2.07132	2.07133	0.00064	0.04937	0.01057	0.00920
83	0.10324	0.13370	2.07630	2.05323	0.00247	0.00750	0.00556	0.00889
84	0.10636	0.12979	2.04735	2.06213	0.00559	0.00359	0.03453	0.00001
85	0.09814	0.11475	2.08094	2.08442	0.00263	0.01146	0.00095	0.02229
86	0.10928	0.09664	2.04605	2.05608	0.00851	0.02956	0.03584	0.00604
87	0.10250	0.12013	2.07445	2.05079	0.00173	0.00607	0.00744	0.01133
88	0.09461	0.14244	2.09829	2.08078	0.00615	0.01624	0.01639	0.01865
89	0.10172	0.11094	2.08401	2.05699	0.00095	0.01526	0.00212	0.00514
90	0.10204	0.11439	2.09723	2.04868	0.00127	0.01181	0.01534	0.01345
91	0.09635	0.15988	2.09498	2.07062	0.00442	0.03368	0.01309	0.00849

92	0.09754	0.11531	2.10781	2.06287	0.00323	0.01089	0.02592	0.00075
93	0.09146	0.10541	2.10223	2.08779	0.00930	0.02080	0.02034	0.02567
94	0.09812	0.09988	2.07513	2.08300	0.00265	0.02634	0.00676	0.02088
95	0.10414	0.14633	2.08264	2.03256	0.00337	0.02013	0.00075	0.02957
96	0.10534	0.14637	2.05595	2.07075	0.00457	0.02016	0.02594	0.00863
97	0.09284	0.14174	2.09640	2.08392	0.00793	0.01554	0.01451	0.02179
98	0.10453	0.12935	2.06802	2.05701	0.00376	0.00315	0.01387	0.00512
99	0.10075	0.14434	2.08353	2.08429	0.00001	0.01814	0.00164	0.02217
100	0.11063	0.09680	2.04975	2.04940	0.00987	0.02942	0.03214	0.01272

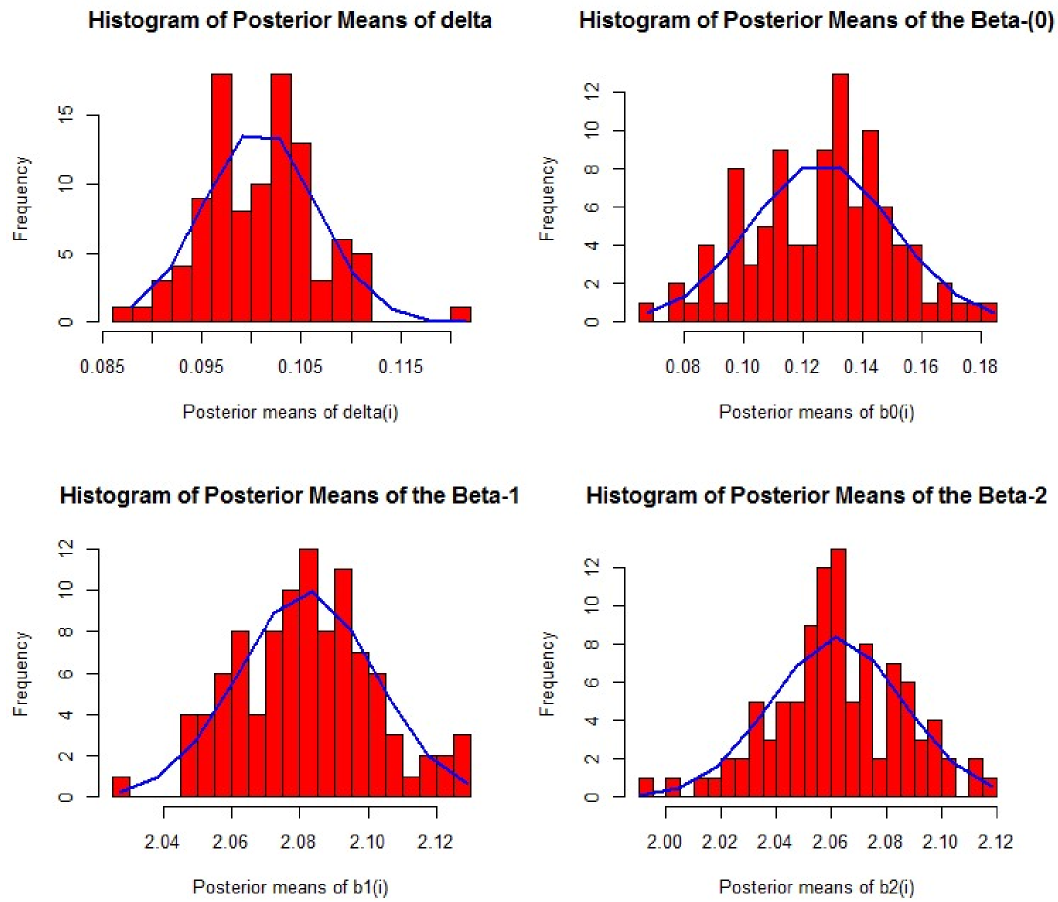

Figure 4.16: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=100 and T=10

Discussion of Results (Tables 4.15 & 4.16 and Figures 4.15 & 4.16)

Table 4.15 exhibits the posterior estimates of second stage hierarchical prior. Here, it is observed that as N increases the posterior means are substantially closer to the initial values even when the error precision was set to be 25. It is obvious that standard deviation of each parameter across all individuals decrease as N tends to large, so also the numerical standard error through the error variance matrix indicates a perfect constant error variance of all the parameters.

It is observed that in Table 4.16 hierarchical Bayesian estimator performed better estimates as second stage hierarchical prior was injected into the first stage hierarchical prior, the posterior estimates gave exact influence of X's over y across all individuals.

Figure 4.15 demonstrates different patterns by posterior means of the simulated datasets while Figure 4.16 projects symmetric shapes for each posterior mean indicating that the datasets are normally distributed.

4.3.3 Performance of Estimator on (N>T): (N=200, T=10)

Table 4.17 The second stage of hierarchical Bayesian Estimation: $\mu_\gamma \mid y, \gamma, h, V_\gamma \sim N(\bar{\mu}_\gamma, \overline{\Sigma}_\gamma)$ When N=200, T=10, h=25, $\beta_{0i} \sim N(0, 0.25)$ $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.02351085	0.05651572	2.12348264	2.1798687	133.4405863
Standard deviation	0.00430891	0.01753861	0.01484233	0.0180112	6.756558950
Numerical Standard Error	0.00013625	0.000554619	0.000469356	0.00056956	0.2136611543

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} \mid y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\bar{v}_\gamma, \left[\bar{v}_\gamma \bar{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.00509 & 0.00003 & -0.00000 & 0.00001 \\ 0.00003 & 0.00507 & 0.00000 & -0.00001 \\ -0.00000 & 0.00000 & 0.00506 & -0.00001 \\ 0.00001 & -0.00001 & 0.00001 & 0.00506 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00053 & 0.00037 & 0.00038 & 0.00037 \\ 0.00037 & 0.00050 & 0.00036 & 0.00037 \\ 0.00038 & 0.00036 & 0.00051 & 0.00036 \\ 0.00037 & 0.00037 & 0.00036 & 0.00051 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00002 & 0.00001 & 0.00001 & 0.00001 \\ 0.00001 & 0.00002 & 0.00001 & 0.00001 \\ 0.00001 & 0.00001 & 0.00002 & 0.00001 \\ 0.00001 & 0.00001 & 0.00001 & 0.00002 \end{bmatrix}$$

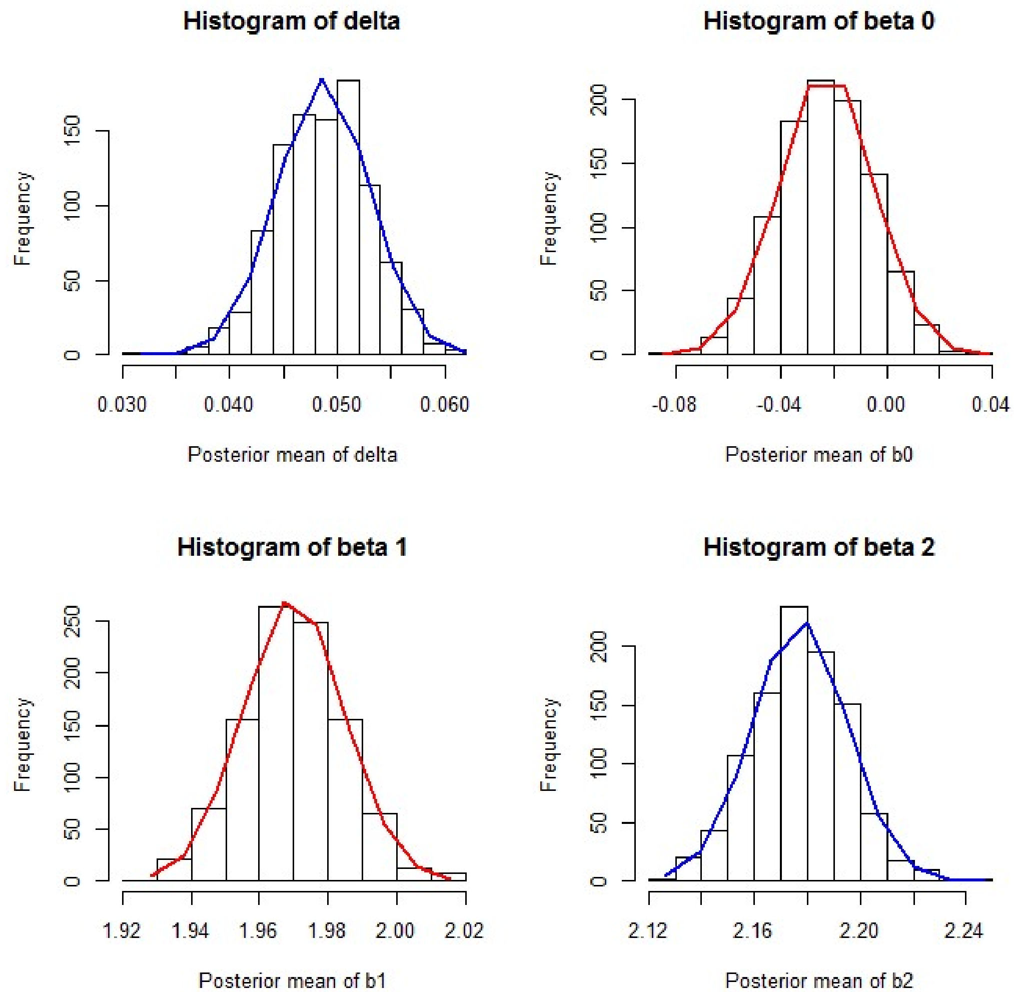

Figure 4.17: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=200, T=10

Table 4.18: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=200, T=10, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior mean				Posterior Standard Deviation			
Ind.	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1.	0.00864	0.05286	2.15850	2.27769	0.01421	0.00371	0.03489	0.04432
2.	0.02122	0.05695	2. 12803	2.17890	0.00163	0.00038	0.00442	0.00440
3.	0.02483	0.05622	2.12213	2.16479	0.00198	0.00035	0.00147	0.01858
4.	0.02070	0.05762	2.13814	2.18397	0.00215	0.00104	0.01454	0.00060
5.	0.02450	0.04689	2.13938	2.16338	0.00164	0.00968	0.01577	0.01998
6.	0.02145	0.05683	2.14245	2.16076	0.00139	0.00025	0.01884	0.02261
7.	0.02106	0.07305	2.11423	2.20678	0.00178	0.01648	0.00937	0.02341
8.	0.02354	0.07175	2.11512	2.18562	0.00068	0.01517	0.00848	0.02246
9.	0.02234	0.04449	2.12392	2.18147	0.00050	0.01208	0.00031	0.00189
10.	0.02344	0.07898	2.12368	2.17681	0.00059	0.02240	0.00007	0.00065
11.	0.02561	0.04474	2.13425	2.16936	0.00276	0.01183	0.01064	0.01401
12.	0.02604	0.05135	2.12935	2.16308	0.00319	0.00521	0.00574	0.02029
13.	0.02463	0.06292	2.11737	2.18651	0.00178	0.00635	0.00623	0.00313
14.	0.02032	0.05372	2.14217	2.17606	0.00252	0.00285	0.01856	0.00731
15.	0.02038	0.08391	2.15966	2.16861	0.00247	0.02734	0.03605	0.01475
16.	0.01794	0.04916	2.12400	2.20165	0.00490	0.00741	0.00004	0.01827
17.	0.01926	0.05436	2.14072	2.17642	0.00359	0.00220	0.01711	0.00695
18.	0.02469	0.07012	2.11507	2.18035	0.00184	0.01354	0.00833	0.00302
19.	0.02365	0.06101	2.13312	2.17691	0.00798	0.00443	0.00951	0.00646
20.	0.02096	0.06077	2.13247	2.19036	0.00189	0.00419	0.00886	0.00698

21.	0.02564	0.05219	2.12540	2.16755	0.00279	0.00437	0.09179	0.01582
22.	0.01905	0.04198	2.14776	2.17285	0.00379	0.01458	0.02415	0.01052
23.	0.02671	0.04616	2.13397	2.14835	0.00386	0.01041	0.01036	0.03501
24.	0.01913	0.05731	2.13984	2.18411	0.00371	0.00074	0.01623	0.00073
25.	0.02658	0.06804	2.11801	2.17521	0.00372	0.01146	0.00558	0.00816
26.	0.02420	0.05896	2.13141	2.17982	0.00134	0.00239	0.00780	0.00355
27.	0.02833	0.03885	2.11424	2.16261	0.00547	0.01772	0.00936	0.02076
28.	0.02962	0.05081	2.11229	2.16323	0.00676	0.00576	0.01131	0.02014
29.	0.02506	0.03972	2.11308	2.18144	0.00221	0.01685	0.01052	0.00193
30.	0.01778	0.02593	2.13496	2.20891	0.00506	0.03063	0.01135	0.02553
31.	0.01707	0.01675	2.13124	2.21075	0.00577	0.03982	0.00763	0.02737
32.	0.02493	0.05495	2.12029	2.17022	0.00208	0.00162	0.00331	0.01314
33.	0.01954	0.07262	2.14262	2.18754	0.00330	0.01605	0.01901	0.00041
34.	0.02145	0.07685	2.13328	2.18322	0.00139	0.02027	0.00967	0.00015
35.	0.01482	0.05028	2.12251	2.21287	0.00802	0.00628	0.00109	0.02950
36.	0.02923	0.05299	2.09812	2.18323	0.00638	0.00357	0.02548	0.00014
37.	0.01959	0.03973	2.12683	2.19267	0.00325	0.01683	0.00322	0.00929
38.	0.01623	0.04619	2.12335	2.22702	0.00661	0.01038	0.00024	0.04364
39.	0.01916	0.07588	2.12420	2.21318	0.00368	0.01930	0.00059	0.02980
40.	0.02667	0.03118	2.09693	2.19176	0.00382	0.02539	0.02666	0.00838
41.	0.02378	0.02818	2.11839	2.18014	0.00093	0.02839	0.00521	0.00323
42.	0.01879	0.05346	2.13458	2.18370	0.00405	0.00311	0.01097	0.00033
43.	0.02012	0.05729	2.13264	2.18141	0.00273	0.00007	0.00903	0.00194
44.	0.02385	0.03164	2.10615	2.18474	0.00100	0.02492	0.01745	0.00137
45.	0.02526	0.03929	2.11286	2.17400	0.00241	0.01727	0.01074	0.00937

46.	0.02862	0.06312	2.11263	2.16212	0.00577	0.00654	0.01097	0.01924
47.	0.02248	0.06084	2.12371	2.19379	0.00036	0.00426	0.00010	0.01041
48.	0.01708	0.04581	2.12672	2.20278	0.00576	0.01076	0.00311	0.01941
49.	0.01972	0.06837	2.11422	2.19764	0.00312	0.01179	0.00938	0.01408
50.	0.02931	0.04484	2.12205	2.17584	0.00646	0.01173	0.00154	0.00753
51.	0.02268	0.07714	2.12457	2.18323	0.00016	0.02056	0.00096	0.00014
52.	0.02694	0.04947	2.11544	2.16850	0.00409	0.00710	0.00816	0.01487
53.	0.02623	0.03810	2.11413	2.17156	0.00338	0.01847	0.00947	0.01181
54.	0.02662	0.00495	2.12402	2.17416	0.00376	0.07008	0.00042	0.00920
55.	0.02388	0.05317	2.14002	2.16119	0.00103	0.00339	0.01641	0.02217
56.	0.02883	0.03638	2.11425	2.18152	0.00597	0.02019	0.00935	0.00185
57.	0.01958	0.03628	2.13411	2.20347	0.00326	0.02029	0.01050	0.02009
58.	0.01773	0.04348	2.14156	2.17939	0.00512	0.01309	0.01795	0.00398
59.	0.02098	0.06862	2.14768	2.16327	0.00186	0.01204	0.02424	0.02010
60.	0.02654	0.01756	2.10494	2.17745	0.00368	0.03901	0.01864	0.00592
61.	0.02653	0.06142	2.12382	2.17296	0.00368	0.00484	0.00021	0.01040
62.	0.02289	0.03313	2.12746	2.18488	0.00438	0.02343	0.00385	0.00150
63.	0.02306	0.05728	2.13022	2.18419	0.00021	0.00070	0.00661	0.00081
64.	0.02027	0.07474	2.11573	2.19612	0.00257	0.01816	0.00787	0.01274
65.	0.01624	0.04408	2.12662	2.19917	0.00661	0.01249	0.00301	0.01579
66.	0.02714	0.04749	2.11019	2.17038	0.00429	0.00908	0.01341	0.01298
67.	0.02098	0.03118	2.11115	2.20039	0.00187	0.02538	0.01245	0.01702
68.	0.02269	0.06140	2.13179	2.18193	0.00015	0.00483	0.00818	0.00144
69.	0.02015	0.05069	2.12871	2.20434	0.00270	0.00587	0.00510	0.02096
70.	0.02255	0.06386	2.13500	2.18398	0.00029	0.00729	0.01140	0.00060

71.	0.02253	0.05056	2.11609	2.19015	0.00031	0.00601	0.00751	0.00677
72.	0.02912	0.05721	2.11255	2.16790	0.00626	0.00063	0.01105	0.01547
73.	0.02043	0.05812	2.13924	2.18055	0.00241	0.00155	0.01563	0.00281
74.	0.01799	0.05051	2.09739	2.21220	0.00485	0.00605	0.02621	0.02882
75.	0.02303	0.05273	2.11273	2.19979	0.00017	0.00383	0.01087	0.01642
76.	0.02531	0.05183	2.11772	2.16824	0.00245	0.00474	0.00588	0.01513
77.	0.02490	0.03783	2.10304	2.18723	0.00205	0.01874	0.02056	0.00385
78.	0.02531	0.07007	2.09723	2.20534	0.00246	0.01350	0.02637	0.02197
79.	0.02599	0.03679	2.11418	2.18167	0.00314	0.01978	0.00942	0.00170
80.	0.02236	0.07398	2.11502	2.18431	0.00048	0.01741	0.00858	0.00093
81.	0.01974	0.06644	2.12139	2.19182	0.00310	0.00986	0.00221	0.00845
82.	0.01925	0.06484	2.11821	2.19983	0.00360	0.00827	0.00539	0.01645
83.	0.02253	0.03895	2.13328	2.17279	0.00031	0.01762	0.00967	0.01058
84.	0.01624	0.04064	2.12491	2.20346	0.00660	0.01593	0.00130	0.02008
85.	0.02213	0.04087	2.08562	2.20597	0.00072	0.01569	0.03798	0.02260
86.	0.02380	0.06576	2.11899	2.19382	0.00094	0.00918	0.00461	0.01044
87.	0.02088	0.05670	2.12865	2.18777	0.00196	0.00012	0.00504	0.00439
88.	0.02211	0.09884	2.13350	2.17114	0.00073	0.04227	0.00989	0.01223
89.	0.01914	0.05267	2.12868	2.19553	0.00370	0.00390	0.00507	0.01215
90.	0.02092	0.08155	2.11414	2.19617	0.00193	0.02498	0.00946	0.01279
91.	0.01422	0.03587	2.16652	2.18219	0.00863	0.02070	0.04291	0.00117
92.	0.02867	0.07345	2.11535	2.14431	0.00581	0.01687	0.00825	0.03906
93.	0.01809	0.05987	2.15429	2.18784	0.00475	0.00329	0.03068	0.00447
94.	0.02936	0.05053	2.11056	2.16464	0.00651	0.00603	0.01304	0.01873
95.	0.02158	0.03835	2.12293	2.19657	0.00127	0.01822	0.00067	0.01320

96.	0.01977	0.09001	2.15632	2.17963	0.00307	0.03343	0.03271	0.00374
97.	0.03071	0.07485	2.11800	2.15570	0.00786	0.01828	0.00560	0.02767
98.	0.02477	0.05395	2.12528	2.17298	0.00192	0.02625	0.00167	0.01038
99.	0.02961	0.08119	2.10736	2.16947	0.00676	0.02461	0.01624	0.01390
100.	0.02049	0.05459	2.14146	2.18183	0.00235	0.00001	0.01785	0.00154
101.	0.02020	0.02833	2.13299	2.18229	0.00264	0.02823	0.00938	0.00108
102.	0.03191	0.07032	2.10861	2.15748	0.00905	0.01374	0.01499	0.02589
103.	0.01681	0.05002	2.12479	2.22418	0.00603	0.00655	0.00118	0.04081
104.	0.01743	0.07162	2.14546	2.18174	0.00542	0.01504	0.02185	0.00163
105.	0.02850	0.05933	2.11699	2.16394	0.00564	0.00276	0.00661	0.01943
106.	0.03436	0.00737	2.08676	2.15765	0.01151	0.01720	0.03684	0.02571
107.	0.02935	0.09744	2.09939	2.16786	0.00650	0.04087	0.02421	0.01550
108.	0.02872	0.08319	2.09870	2.17022	0.00587	0.02662	0.02490	0.01314
109.	0.02099	0.06776	2.13557	2.17343	0.00185	0.01119	0.01196	0.00994
110.	0.02460	0.06338	2.12876	2.18523	0.00175	0.00681	0.00515	0.00186
111.	0.02190	0.05051	2.15263	2.17395	0.00095	0.00605	0.02902	0.00942
112.	0.01835	0.06452	2.14358	2.17583	0.00449	0.00795	0.01997	0.00754
113.	0.01872	0.08782	2.12544	2.20762	0.00412	0.03124	0.00183	0.02425
114.	0.02030	0.08620	2.21177	2.19817	0.00254	0.02963	0.00582	0.01479
115.	0.02246	0.05744	2.09159	2.19893	0.00038	0.00086	0.03201	0.01555
116.	0.02365	0.06781	2.11714	2.17898	0.00079	0.01123	0.00646	0.00439
117.	0.03383	0.05183	2.07931	2.16713	0.01098	0.00474	0.04429	0.01623
118.	0.02574	0.07617	2.12015	2.17602	0.00289	0.01960	0.00345	0.00735
119.	0.02564	0.04248	2.10374	2.19209	0.00278	0.01409	0.01986	0.00872
120.	0.01853	0.05756	2.14277	2.19515	0.00431	0.00098	0.01916	0.01177
121.	0.01416	0.07017	2.13750	2.21181	0.00868	0.01359	0.01389	0.02843
122.	0.01261	0.03896	2.13644	2.21904	0.01023	0.01760	0.01283	0.03567
123.	0.02605	0.05199	2.11336	2.15972	0.00320	0.00045	0.01024	0.02364
124.	0.02787	0.07710	2.10458	2.18029	0.00502	0.00205	0.01902	0.00307

125.	0.01619	0.04092	2.13964	2.20665	0.00665	0.01565	0.01603	0.02327
126.	0.01585	0.05233	2.14312	2.20436	0.00699	0.00424	0.09151	0.02099
127.	0.02315	0.05836	2.13264	2.17656	0.00030	0.00179	0.09034	0.00680
128.	0.02647	0.05001	2.10531	2.18798	0.00362	0.00656	0.01829	0.00460
129.	0.02299	0.02759	2.13524	2.18249	0.00014	0.02897	0.01633	0.00092
130.	0.02437	0.05359	2.09633	2.20434	0.00152	0.00297	0.02727	0.02096
131.	0.02075	0.04945	2.10685	2.20838	0.00209	0.00711	0.01675	0.02500
132.	0.02290	0.06659	2.11452	2.18140	0.00056	0.01001	0.00908	0.00197
133.	0.01980	0.06234	2.14417	2.19007	0.00305	0.00576	0.02056	0.00869
134.	0.02898	0.05362	2.12660	2.14214	0.00613	0.00295	0.00299	0.04123
135.	0.02566	0.04431	2.13014	2.16388	0.00281	0.01226	0.00653	0.01948
136.	0.02060	0.08410	2.13381	2.16982	0.00224	0.02752	0.01020	0.01355
137.	0.01851	0.05595	2.11994	2.20777	0.00433	0.00061	0.00366	0.02439
138.	0.02086	0.04553	2.12041	2.19247	0.00199	0.01104	0.00319	0.00910
139.	0.02163	0.03844	2.12148	2.17595	0.00121	0.01813	0.00212	0.00742
140.	0.02487	0.08119	2.13450	2.15644	0.00202	0.02462	0.01089	0.02693
141.	0.02081	0.05015	2.13360	2.18005	0.00203	0.00642	0.00999	0.00332
142.	0.01827	0.04301	2.13900	2.19260	0.00458	0.01356	0.01539	0.00923
143.	0.02497	0.01245	2.11750	2.17219	0.00211	0.06794	0.00610	0.01117
144.	0.02738	0.05795	2.10097	2.18502	0.00452	0.00138	0.02262	0.00165
145.	0.01820	0.05314	2.15236	2.18306	0.00465	0.00343	0.02876	0.00031
146.	0.02009	0.06525	2.13609	2.18621	0.00275	0.00867	0.01248	0.00283
147.	0.02914	0.06010	2.11226	2.17813	0.00629	0.00352	0.01134	0.00524
148.	0.02311	0.01877	2.12044	2.17355	0.00026	0.03779	0.00136	0.00981
149.	0.01808	0.05727	2.12891	2.19461	0.00476	0.00069	0.00530	0.01123
150.	0.02945	0.04761	2.09522	2.16047	0.00659	0.00895	0.02838	0.02290
151.	0.02456	0.04460	2.12537	2.16318	0.00171	0.01197	0.00176	0.02019
152.	0.02172	0.05399	2.13054	2.18701	0.00112	0.00257	0.00693	0.00364
153.	0.01469	0.04985	2.15279	2.20928	0.00815	0.00672	0.02918	0.02590
154.	0.026463	0.04793	2.11261	2.17021	0.00360	0.00863	0.01099	0.01315
155.	0.016177	0.05884	2.15448	2.18112	0.00668	0.00226	0.03087	0.00225

156.	0.028041	0.06224	2.09989	2.16934	0.00518	0.00567	0.02371	0.01403
157.	0.021017	0.04902	2.13246	2.18447	0.00184	0.00755	0.00886	0.00109
158.	0.022086	0.05615	2.12000	2.20237	0.00077	0.00042	0.00360	0.01899
159.	0.029074	0.03503	2.10541	2.15802	0.00622	0.02153	0.01819	0.02534
160.	0.017996	0.08367	2.12862	2.20662	0.00485	0.02710	0.00502	0.02325
161.	0.024794	0.04360	2.15310	2.14318	0.00194	0.01297	0.02949	0.04019
162.	0.020598	0.06734	2.12514	2.18860	0.00226	0.01076	0.00153	0.00523
163.	0.02344	0.06984	2.10930	2.19229	0.00058	0.01326	0.01430	0.00891
164.	0.01134	0.05525	2.15481	2.20911	0.01151	0.00131	0.03120	0.02573
165.	0.02680	0.06726	2.09758	2.18126	0.00397	0.01069	0.02602	0.00211
166.	0.02447	0.06308	2.10338	2.18854	0.00169	0.00650	0.02022	0.00516
167.	0.02362	0.06431	2.12544	2.16260	0.00077	0.00773	0.01840	0.02076
168.	0.01434	0.07194	2.13669	2.21629	0.00850	0.01536	0.01308	0.03291
169.	0.01950	0.07937	2.13110	2.19877	0.00335	0.02280	0.00749	0.01540
170.	0.02669	0.04635	2.11349	2.18517	0.00384	0.01021	0.01011	0.00179
171.	0.02892	0.08235	2.09709	2.19160	0.00607	0.02577	0.02651	0.00822
172.	0.02584	0.06314	2.11813	2.16998	0.00299	0.00657	0.05474	0.01338
173.	0.02018	-0.0020	2.13482	2.18115	0.00267	0.05861	0.01121	0.00222
174.	0.01804	0.07627	2.13071	2.19448	0.00480	0.01969	0.00710	0.01110
175.	0.02316	0.04520	2.12740	2.15436	0.00031	0.01137	0.00379	0.02900
176.	0.02675	0.05122	2.12101	2.17202	0.00389	0.00535	0.00259	0.01134
177.	0.02646	0.04125	2.10719	2.20700	0.00360	0.01531	0.01641	0.02362
178.	0.01833	0.06014	2.14176	2.20695	0.00451	0.00357	0.01815	0.02357
179.	0.02584	0.07327	2.10969	2.19086	0.00299	0.01669	0.01391	0.00749
180.	0.02550	0.05333	2.11767	2.18988	0.00265	0.00323	0.00593	0.00650
181.	0.03276	0.08785	2.09195	2.16501	0.00990	0.03128	0.03165	0.01835
182.	0.01584	0.08178	2.14066	2.21542	0.00700	0.02511	0.01705	0.03205
183.	0.02676	0.05613	2.10815	2.18450	0.00390	0.00044	0.01545	0.00113
184.	0.02504	0.05388	2.12415	2.18434	0.00218	0.02695	0.00054	0.00096
185.	0.03238	0.06588	2.08602	2.15989	0.00953	0.00931	0.03758	0.02348
186.	0.02549	0.04439	2.14598	2.15532	0.00264	0.01218	0.02238	0.02804

187.	0.02555	0.07672	2.13199	2.16480	0.00270	0.02014	0.00838	0.01856
188.	0.02808	0.05653	2.10896	2.18121	0.00522	0.00003	0.01464	0.00216
189.	0.02450	0.03749	2.11527	2.17750	0.00165	0.01907	0.00833	0.00586
190.	0.01987	0.08411	2.13121	2.20759	0.00366	0.02753	0.00760	0.02421
191.	0.02289	0.07038	2.11483	2.17173	0.00143	0.01380	0.00877	0.01163
192.	0.02809	0.06210	2.12661	2.16016	0.00523	0.00552	0.00301	0.02320
193.	0.02039	0.07103	2.14428	2.18238	0.00246	0.01445	0.02067	0.00099
194.	0.02389	0.06171	2.11615	2.18594	0.00104	0.00514	0.00745	0.00256
195.	0.02038	0.03698	2.12787	2.19295	0.00246	0.01959	0.00427	0.00958
196.	0.02657	0.06768	2.10991	2.16846	0.00372	0.01104	0.01369	0.01491
197.	0.02467	0.05276	2.12858	2.19087	0.00181	0.00380	0.00497	0.00749
198.	0.02119	0.04529	2.11572	2.19921	0.00166	0.01128	0.00787	0.01583
199.	0.02130	0.03274	2.12013	2.20098	0.00154	0.02383	0.00347	0.01760
200.	0.02262	0.03470	2.12964	2.16614	0.00022	0.02186	0.00603	0.01723

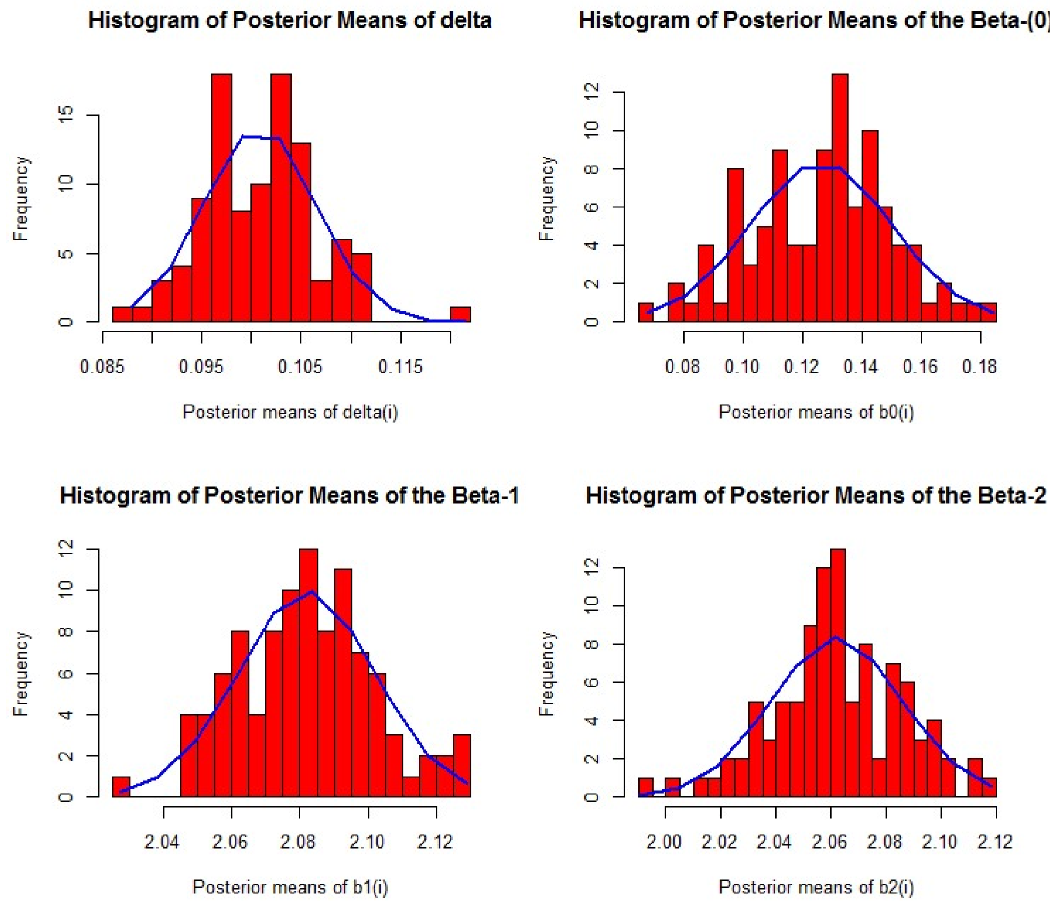

Figure 4.18: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=200 and T=10

4.3.4 Performance of Estimator on (N>T): (N=200, T=20)

Table 4.19: The second stage of hierarchical Bayesian estimates: $\mu_\gamma \mid y,\gamma,h,V_\gamma \sim N(\bar{\mu}_\gamma,\bar{\Sigma}_\gamma)$ When N=200, T=20, h=25, β_{0i} ~ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.09538802	0.19013004	2.04777405	2.16638865	247.02428
Standard deviation	0.00309614	0.01251531	0.010898460	0.012709515	9.145090868
Numerical Standard Error	0.00009790	0.000395768	0.000344639	0.000401910	0.28919322

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} \mid y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\bar{v}_\gamma,\left[\bar{v}_\gamma \bar{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.00507 & 0.00000 & 0.00000 & -0.00000 \\ 0.00000 & 0.00506 & -0.00002 & 0.00001 \\ 0.00000 & -0.00002 & 0.00507 & -0.00000 \\ -0.00000 & 0.00001 & -0.00000 & 0.00507 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00050 & 0.00036 & 0.00036 & 0.00036 \\ 0.00036 & 0.00052 & 0.00036 & 0.00036 \\ 0.00036 & 0.00036 & 0.00052 & 0.00035 \\ 0.00036 & 0.00036 & 0.00035 & 0.00050 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00002 & 0.00001 & 0.00001 & 0.00001 \\ 0.00001 & 0.00002 & 0.00001 & 0.00001 \\ 0.00001 & 0.00001 & 0.00002 & 0.00001 \\ 0.00001 & 0.00001 & 0.00001 & 0.00002 \end{bmatrix}$$

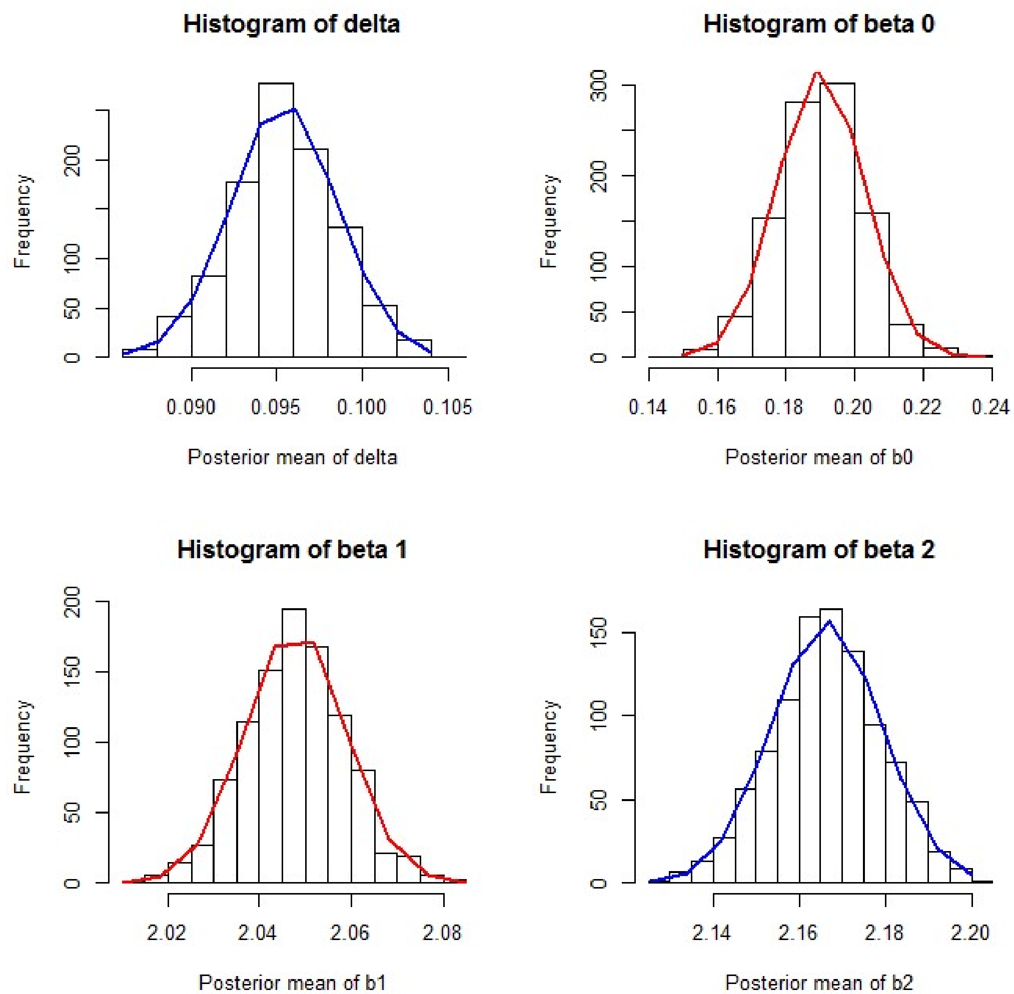

Figure 4.19: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=200, T=20

Table 4.20: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i \mid y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=200, T=20, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior Mean				Posterior Standard Deviation			
	δ	β0	β1	β2	δ	β0	β1	β2
1	0.09644	0.18720	2.04471	2.15816	0.00110	0.00291	0.00415	0.00779
2	0.09473	0.18100	2.05196	2.16389	0.00059	0.00911	0.00302	0.00205
3	0.09678	0.18441	2.05087	2.16474	0.00144	0.00571	0.00193	0.00121
4	0.09537	0.19621	2.06135	2.15319	0.00031	0.00060	0.01241	0.01275
5	0.09451	0.19804	2.05312	2.16433	0.00082	0.00791	0.00418	0.00161
6	0.09194	0.18711	2.04910	2.18474	0.00339	0.00301	0.00017	0.01879
7	0.09627	0.20834	2.05347	2.15605	0.00093	0.01821	0.00454	0.00989
8	0.09305	0.20488	2.05870	2.16298	0.00228	0.01475	0.00977	0.00279
9	0.10115	0.20061	2.03271	2.13803	0.00581	0.01048	0.01622	0.02791
10	0.09807	0.18470	2.03859	2.16243	0.00273	0.00542	0.01033	0.00355
11	0.09769	0.18075	2.04648	2.15065	0.00235	0.00937	0.00245	0.01529
12	0.10107	0.18710	2.03751	2.15296	0.00573	0.00302	0.01141	0.01298
13	0.09766	0.17383	2.05086	2.15324	0.00232	0.01639	0.00192	0.01270
14	0.09939	0.20491	2.03651	2.15723	0.00405	0.01388	0.01241	0.00871
15	0.09794	0.21081	2.04224	2.16183	0.00260	0.02068	0.00669	0.00412
16	0.09485	0.21120	2.04583	2.16853	0.00048	0.02107	0.00309	0.00258
17	0.09711	0.19119	2.06453	2.14879	0.00177	0.00106	0.01560	0.01716
18	0.09583	0.18204	2.05152	2.16680	0.00049	0.00808	0.00259	0.00085
19	0.09200	0.20200	2.05301	2.18467	0.00333	0.01187	0.00407	0.01872
20	0.09147	0.19274	2.06462	2.17279	0.00386	0.00261	0.00156	0.00684
21	0.09049	0.19591	2.05173	2.19054	0.00484	0.00578	0.00279	0.02459
22	0.09831	0.18819	2.03877	2.16252	0.00297	0.00193	0.01016	0.00342
23	0.09029	0.19357	2.06706	2.18046	0.00503	0.00344	0.01812	0.01451
24	0.09519	0.19804	2.06368	2.16260	0.00014	0.00791	0.01474	0.00334
25	0.09575	0.18304	2.03622	2.16461	0.00042	0.00708	0.01271	0.00133
26	0.09644	0.16605	2.04679	2.16383	0.00110	0.02407	0.00214	0.00212
27	0.09973	0.18874	2.04959	2.13904	0.00439	0.00137	0.00006	0.02690

28	0.09803	0.19536	2.01445	2.18349	0.00269	0.00523	0.03448	0.01754
29	0.09161	0.19399	2.06154	2.17494	0.00272	0.00386	0.01261	0.00899
30	0.09954	0.21248	2.03688	2.16081	0.00420	0.02235	0.01205	0.00513
31	0.08898	0.17551	2.06691	2.18811	0.00635	0.01461	0.01797	0.02215
32	0.09850	0.17465	2.03350	2.17042	0.00316	0.01547	0.01542	0.00447
33	0.09679	0.18638	2.04552	2.16683	0.00146	0.00374	0.00340	0.00088
34	0.08743	0.18416	2.07484	2.18388	0.00790	0.00596	0.02590	0.01793
35	0.09846	0.19746	2.08235	2.16857	0.00312	0.00733	0.02057	0.00262
36	0.09572	0.19475	2.06193	2.15275	0.00038	0.00462	0.01299	0.01320
37	0.09447	0.20498	2.04371	2.7422	0.00085	0.01485	0.00522	0.00827
38	0.09275	0.19806	2.05819	2.16916	0.00258	0.00793	0.00925	0.00322
39	0.09068	0.18217	2.06138	2.17977	0.00465	0.00795	0.01244	0.01378
40	0.10330	0.21184	2.02609	2.13950	0.00796	0.02171	0.02283	0.02636
41	0.10129	0.18722	2.03784	2.14302	0.00596	0.00290	0.01109	0.02287
42	0.09665	0.17706	2.03699	2.17223	0.00132	0.01306	0.01194	0.00634
43	0.10146	0.21146	2.04073	2.13649	0.00612	0.02133	0.00819	0.02954
44	0.09764	0.19572	2.05606	2.14879	0.00230	0.00559	0.00713	0.01716
45	0.09680	0.16222	2.04228	2.16446	0.00147	0.02789	0.00665	0.00148
46	0.09197	0.19009	2.05325	2.17460	0.00336	0.00003	0.00432	0.00865
47	0.09467	0.18371	2.04496	2.16899	0.00066	0.00641	0.00396	0.00304
48	0.09733	0.19108	2.04243	2.16018	0.00200	0.00095	0.00650	0.05771
49	0.09371	0.18999	2.04231	2.17487	0.00161	0.00013	0.00662	0.00892
50	0.09624	0.18952	2.04145	2.16614	0.00090	0.00059	0.00748	0.00019
51	0.10068	0.19755	2.03835	2.15118	0.00534	0.00743	0.01058	0.01476
52	0.09188	0.18685	2.06646	2.17166	0.00345	0.00326	0.01752	0.00570
53	0.08797	0.19052	2.05344	2.20110	0.00736	0.00039	0.00450	0.03515
54	0.09271	0.20954	2.04375	2.18694	0.00262	0.01941	0.00518	0.02098
55	0.09516	0.17976	2.03882	2.17920	0.00017	0.01036	0.01010	0.01325
56	0.09229	0.16782	2.06729	2.17111	0.00304	0.02230	0.01835	0.00515
57	0.09318	0.18087	2.05690	2.17240	0.00215	0.00924	0.00796	0.00645
58	0.09728	0.20739	2.05808	2.14580	0.00194	0.01726	0.00914	0.02015
59	0.09521	0.19733	2.05251	2.15949	0.00012	0.00720	0.00358	0.00645
60	0.09942	0.19421	2.03679	2.16310	0.00408	0.00409	0.01214	0.00284

61	0.09544	0.20083	2.06194	2.16586	0.00011	0.010708	0.01306	0.00009
62	0.09636	0.17301	2.04767	2.15177	0.00102	0.01712	0.00129	0.01418
63	0.08885	0.19928	2.07348	2.18091	0.00648	0.00915	0.02455	0.01496
64	0.09847	0.20170	2.04841	2.15505	0.00319	0.01150	0.00052	0.01089
65	0.09386	0.19510	2.06214	2.15816	0.00147	0.00497	0.01321	0.00778
66	0.09090	0.18274	2.05448	2.17805	0.00443	0.00738	0.05545	0.01210
67	0.09529	0.19758	2.06335	2.15448	0.00004	0.00745	0.00144	0.01146
68	0.09598	0.16239	2.04894	2.16098	0.00065	0.02773	0.00009	0.00497
69	0.09783	0.20106	2.04459	2.15233	0.00249	0.01093	0.00434	0.01362
70	0.09198	0.18881	2.03818	2.18468	0.003351	0.00131	0.01075	0.01873
71	0.09556	0.18286	2.04553	2.16580	0.00028	0.00726	0.00339	0.00015
72	0.09783	0.16771	2.04565	2.14912	0.00247	0.02241	0.00328	0.01682
73	0.09490	0.19283	2.04512	2.17963	0.00045	0.00270	0.00381	0.01367
74	0.09232	0.19070	2.04550	2.17820	0.00306	0.00058	0.00557	0.01225
75	0.09435	0.19813	2.05446	2.16422	0.00095	0.00800	0.00552	0.00172
76	0.09739	0.17499	2.04027	2.15199	0.00204	0.01513	0.00866	0.01396
77	0.09174	0.19140	2.06425	2.17847	0.00353	0.00127	0.01532	0.01251
78	0.09584	0.19476	2.04336	2.16797	0.00055	0.00463	0.00557	0.00201
79	0.09578	0.18717	2.04675	2.15785	0.00047	0.00295	0.00217	0.00809
80	0.09802	0.18786	2.04333	2.15734	0.00284	0.00226	0.00560	0.00860
81	0.09591	0.19741	2.05083	2.16766	0.00073	0.00728	0.00190	0.00171
82	0.09428	0.19133	2.05463	2.15871	0.00148	0.00120	0.00569	0.00723
83	0.10220	0.18389	2.03080	2.13872	0.00862	0.00623	0.01812	0.02722
84	0.09064	0.18172	2.05065	2.19744	0.00696	0.00840	0.00171	0.03149
85	0.09595	0.19374	2.04360	2.16800	0.00616	0.00361	0.00532	0.00204
86	0.09921	0.19574	2.02650	2.16175	0.00880	0.00561	0.02243	0.00420
87	0.09375	0.21468	2.04915	2.17538	0.00581	0.02455	0.00021	0.00943
88	0.09136	0.21120	2.05654	2.18070	0.00975	0.02107	0.00760	0.01474
89	0.09400	0.20602	2.05427	2.16180	0.00330	0.01590	0.00533	0.00414
90	0.09548	0.19179	2.04640	2.17051	0.00142	0.00166	0.00253	0.00456
91	0.09795	0.20224	2.03380	2.16545	0.00614	0.01211	0.01512	0.00050
92	0.09143	0.20697	2.03912	2.19100	0.00901	0.01684	0.00980	0.02505
93	0.09272	0.18917	2.05965	2.17558	0.00617	0.00095	0.01071	0.00963

94	0.09212	0.17584	2.05836	2.17588	0.00210	0.01428	0.00942	0.00993
95	0.08930	0.17401	2.07535	2.16900	0.00035	0.01610	0.02642	0.00304
96	0.10331	0.18882	2.05439	2.13970	0.00978	0.00130	0.00353	0.02625
97	0.09879	0.19884	2.04718	2.14881	0.00460	0.00872	0.00175	0.01713
98	0.09606	0.19000	2.04923	2.15779	0.00727	0.00012	0.00029	0.00815
99	0.09557	0.18764	2.03136	2.16757	0.00239	0.00248	0.00175	0.00162
100	0.09136	0.19603	2.05134	2.17349	0.00977	0.00591	0.00241	0.00754
101	0.09763	0.18670	2.03860	2.16568	0.00294	0.00342	0.01032	0.00026
102	0.09705	0.19277	2.05517	2.15970	0.00720	0.00264	0.00623	0.00624
103	0.08971	0.20655	2.06147	2.18071	0.00621	0.01642	0.00125	0.01476
104	0.09520	0.17504	2.05680	2.16402	0.00129	0.01508	0.00786	0.00192
105	0.09234	0.18050	2.04575	2.17738	0.00995	0.00962	0.00318	0.01188
106	0.09936	0.19457	2.03407	2.15524	0.00027	0.00444	0.01485	0.01070
107	0.09231	0.19016	2.04522	2.18722	0.00021	0.00003	0.00370	0.02127
108	0.04979	0.18885	2.05261	2.16009	0.00039	0.00127	0.00368	0.00585
109	0.09906	0.18714	2.03151	2.16231	0.00373	0.00298	0.01741	0.00364
110	0.08992	0.17319	2.07033	2.17845	0.00540	0.01693	0.02139	0.01249
111	0.09204	0.17880	2.04728	2.19235	0.00329	0.01132	0.00164	0.02635
112	0.09781	0.16091	2.04554	2.16571	0.00247	0.02920	0.00339	0.00023
113	0.08838	0.17935	2.05762	2.19180	0.00694	0.01077	0.00869	0.02585
114	0.09616	0.19518	2.03358	2.16920	0.00082	0.00505	0.01534	0.00325
115	0.09289	0.20605	2.05100	2.17337	0.00244	0.01592	0.00207	0.00742
116	0.09632	0.17890	2.04817	2.15659	0.00098	0.01122	0.00075	0.00935
117	0.09405	0.18176	2.05392	2.16894	0.00128	0.00836	0.00499	0.00299
118	0.10247	0.16898	2.03070	2.14399	0.00713	0.02114	0.01226	0.02195
119	0.09817	0.20944	2.04032	2.16285	0.00283	0.01931	0.00860	0.00310
120	0.09350	0.18055	2.05833	2.16999	0.00183	0.00957	0.00939	0.00402
121	0.09799	0.17908	2.02588	2.16991	0.00265	0.01103	0.02304	0.00396
122	0.10000	0.18448	2.04279	2.15324	0.00464	0.00564	0.00614	0.01270
123	0.09789	0.17568	2.04284	2.16703	0.00256	0.01444	0.00608	0.00107
124	0.09550	0.18493	2.05479	2.15977	0.00016	0.00519	0.00586	0.00617
125	0.09223	0.18530	2.05979	2.15676	0.00309	0.00482	0.01085	0.00173
126	0.09767	0.17354	2.04346	2.16393	0.00234	0.01658	0.00547	0.00203

127	0.09812	0.17682	2.04592	2.16117	0.00278	0.01329	0.00300	0.00475
128	0.09499	0.19982	2.04052	2.16933	0.00034	0.00969	0.00840	0.00340
129	0.09801	0.17765	2.05888	2.13458	0.00267	0.01247	0.00994	0.03140
130	0.09095	0.19211	2.05393	2.18300	0.00438	0.00198	0.05002	0.01711
131	0.09561	0.19356	2.04566	2.17340	0.00027	0.00343	0.00327	0.00745
132	0.09675	0.19447	2.04074	2.17086	0.00141	0.00434	0.00818	0.00491
133	0.09506	0.20099	2.03983	2.16934	0.00027	0.01086	0.00909	0.00338
134	0.09416	0.20895	2.05214	2.16066	0.00117	0.01882	0.00321	0.00529
135	0.09278	0.18990	2.06240	2.16785	0.00255	0.00022	0.01346	0.00190
136	0.09444	0.19692	2.05038	2.16422	0.00089	0.00679	0.00144	0.00172
137	0.09618	0.18210	2.05494	2.16211	0.00085	0.00802	0.00601	0.00383
138	0.10027	0.17065	2.03184	2.16135	0.00494	0.01947	0.01708	0.00459
139	0.09415	0.19033	2.04564	2.16948	0.00118	0.00020	0.00329	0.00353
140	0.09877	0.17653	2.03915	2.15462	0.03433	0.01359	0.00978	0.01133
141	0.08865	0.17717	2.04536	2.20307	0.00668	0.01295	0.00357	0.03712
142	0.09290	0.19388	2.06517	2.16737	0.00243	0.00375	0.01621	0.00142
143	0.09232	0.19131	2.05906	2.18053	0.00300	0.00118	0.01013	0.01458
144	0.08861	0.19023	2.07294	2.18555	0.00672	0.00010	0.02400	0.01960
145	0.09365	0.18888	2.05489	2.16899	0.00168	0.00124	0.00595	0.00304
146	0.09336	0.18289	2.05132	2.17082	0.00197	0.00723	0.00239	0.00486
147	0.09813	0.19867	2.03454	2.16984	0.00279	0.00854	0.01438	0.00389
148	0.09424	0.18144	2.06285	2.15403	0.00109	0.00868	0.01392	0.01191
149	0.09496	0.18627	2.04179	2.17117	0.00037	0.00385	0.00713	0.00521
150	0.09180	0.19499	2.04547	2.18224	0.00353	0.04863	0.00345	0.01629
151	0.09536	0.18345	2.04493	2.16803	0.00002	0.00667	0.00400	0.00208
152	0.09786	0.19600	2.04987	2.14667	0.00252	0.00587	0.00093	0.01928
153	0.09282	0.17510	2.05204	2.17512	0.00251	0.01502	0.00311	0.00917
154	0.09173	0.20342	2.06864	2.17112	0.00360	0.01329	0.01970	0.00517
155	0.09844	0.19441	2.04352	2.15336	0.03103	0.00428	0.00540	0.01258
156	0.09859	0.18118	2.03655	2.16012	0.00326	0.00894	0.01238	0.00583
157	0.08917	0.18334	2.07346	2.18599	0.00124	0.00062	0.02452	0.02003
158	0.09658	0.19075	2.05122	2.15533	0.00160	0.00274	0.00229	0.01061
159	0.09373	0.18738	2.04108	2.17674	0.00353	0.04863	0.00784	0.01079

160	0.09107	0.19816	2.07228	2.15939	0.00426	0.00803	0.02335	0.00656
161	0.09696	0.21035	2.03287	2.16773	0.00162	0.02022	0.01606	0.00177
162	0.09656	0.20000	2.05226	2.15356	0.00122	0.00987	0.00333	0.01238
163	0.09960	0.20651	2.04365	2.15243	0.00426	0.01638	0.00527	0.01351
164	0.07551	0.21677	2.05638	2.13541	0.00221	0.02664	0.00744	0.03054
165	0.09322	0.19942	2.06704	2.16308	0.00211	0.00930	0.01810	0.00286
166	0.09754	0.18034	2.05952	2.14830	0.00221	0.00978	0.01059	0.01764
167	0.09393	0.19833	2.04243	2.17743	0.00140	0.00820	0.00649	0.01148
168	0.09294	0.17190	2.04225	2.18538	0.00239	0.01822	0.00667	0.01943
169	0.09829	0.19052	2.04343	2.16188	0.00295	0.00039	0.00550	0.00406
170	0.09774	0.19465	2.02984	2.16861	0.00240	0.00452	0.01909	0.00265
171	0.09781	0.19580	2.04684	2.16429	0.00247	0.00567	0.00209	0.00165
172	0.09397	0.18449	2.05785	2.17505	0.00135	0.00563	0.00892	0.00909
173	0.09728	0.18606	2.04499	2.15466	0.00194	0.00406	0.00394	0.01128
174	0.09191	0.20378	2.05860	2.17821	0.00342	0.01365	0.00967	0.01225
175	0.09453	0.19605	2.06292	2.16307	0.00080	0.00592	0.01399	0.00287
176	0.09305	0.22387	2.04979	2.17731	0.00228	0.03374	0.00086	0.01136
177	0.09247	0.19018	2.05344	2.16740	0.00286	0.00005	0.00450	0.00144
178	0.09461	0.20553	2.04458	2.17353	0.00072	0.01540	0.00435	0.00758
179	0.09359	0.17610	2.04803	2.17518	0.00174	0.01402	0.00090	0.00923
180	0.09731	0.17096	2.03778	2.16226	0.00197	0.01916	0.01115	0.00368
181	0.09503	0.20391	2.03107	2.17721	0.00030	0.01379	0.01785	0.01126
182	0.09752	0.17416	2.04034	2.16071	0.00218	0.01596	0.00859	0.00524
183	0.09314	0.20449	2.04775	2.17061	0.00219	0.01436	0.00117	0.00466
184	0.09612	0.19661	2.03175	2.18403	0.00078	0.00648	0.01717	0.01807
185	0.09877	0.18491	2.04211	2.16013	0.00343	0.00521	0.00682	0.00581
186	0.09800	0.18759	2.05140	2.14791	0.00266	0.00253	0.00246	0.01803
187	0.09861	0.20074	2.03135	2.15443	0.00327	0.01061	0.01758	0.01151
188	0.10020	0.19999	2.05657	2.13505	0.00486	0.00986	0.00763	0.03090
189	0.09657	0.18838	2.04441	2.15886	0.00123	0.00174	0.00451	0.00708
190	0.09717	0.20202	2.05069	2.16401	0.00183	0.01899	0.00176	0.00193
191	0.09560	0.18509	2.05552	2.16269	0.00026	0.00503	0.00659	0.00325
192	0.09500	0.18319	2.03893	2.17302	0.00033	0.00693	0.01001	0.00706

193	0.09839	0.18078	2.04041	2.15401	0.00305	0.00934	0.00852	0.01193
194	0.09402	0.16009	2.05929	2.17264	0.00131	0.03003	0.01035	0.00669
195	0.09287	0.18408	2.04988	2.18034	0.00246	0.00604	0.09541	0.01439
196	0.09545	0.20263	2.05563	2.15580	0.00011	0.01250	0.00669	0.01014
197	0.09134	0.18735	2.06482	2.16892	0.00399	0.00277	0.01589	0.00269
198	0.09371	0.18260	2.06524	2.16454	0.00162	0.00752	0.01649	0.00141
199	0.09782	0.18127	2.05217	2.15622	0.00248	0.00885	0.00323	0.00973
200	0.09629	0.17375	2.05952	2.15694	0.00095	0.01637	0.01058	0.00900

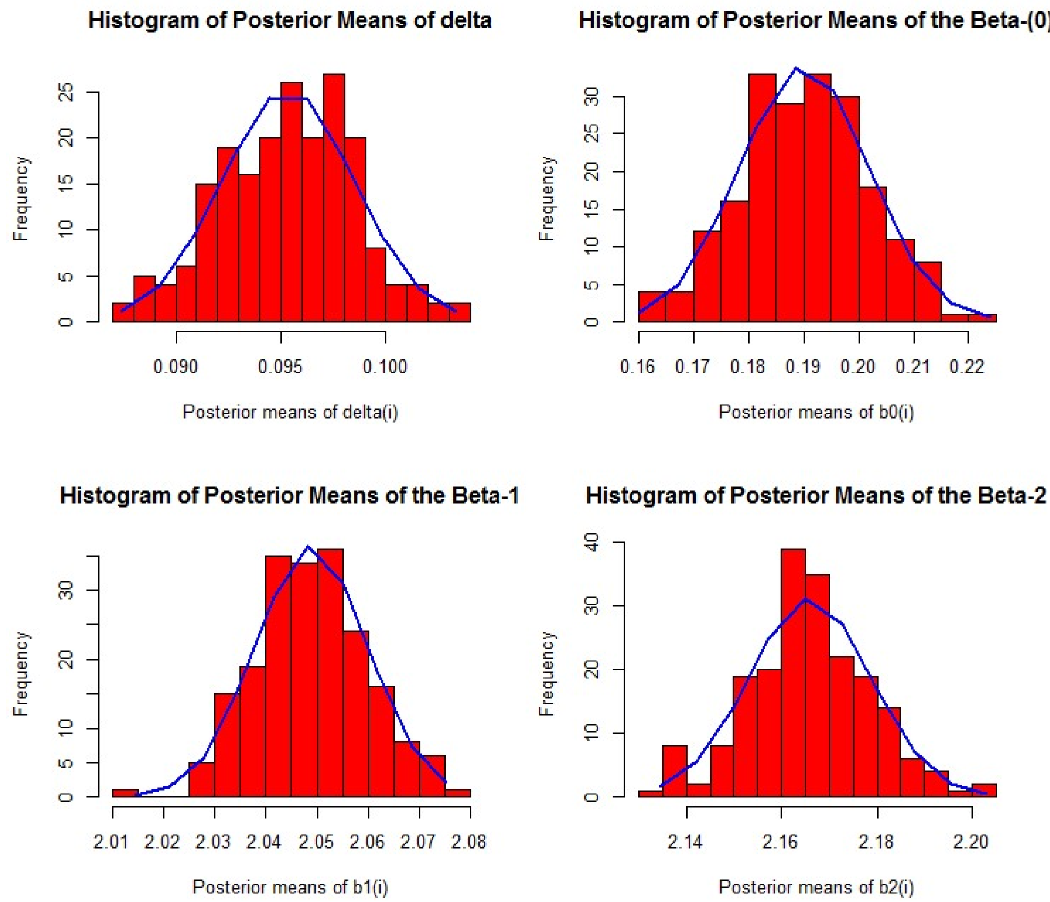

Figure 4.20: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=200 and T=20

Discussion of Results (Tables 4.17 & 4.19 and Figures 4.17 & 4.19)

Table 4.17 and Table 4.19present the posterior estimates of second stage hierarchical prior for $\mu_\gamma(\delta, \beta_0, \beta_1, \beta_2)$. The posterior means for all parameters are unbiased and consistent with the initial values of every sample size considered. δ_i are non-negative and β_{0i} are given a positive posterior estimate all throughwhile β_{1i} and β_{2i} approach the true values consistently. MCMC diagnostics indicate convergence of all the Gibbs samplers and numerical standard errors indicate an approximation error which is small relative to posterior standard deviations of all parameters. Figure 4.17 and 4.19 look to be replicating the same pattern of previous graphs quite accurately.

An examination of Figure 4.18 and 4.20 shows that random coefficients model is very appropriate at picking out the variation in the parameters. Hence, hierarchical Bayesian estimator is found useful in locating complicated patterns embedded in the data. It is important to model variation in the parameter correctly so that such variation will not be assigned to error variance in order to avoid misleading inferences with regard to coefficients.

4.4 Investigation of Sensitivity of Prior Information on the Posterior Estimates of Heterogeneous Dynamic Panel Data Model: Relatively Non- Informative and Informative Prior

4.4.1 The Prior Information on Posterior Estimates

The prior distributionis a major component of Bayesian estimation which represents the information about the unknown parameters combined with the probability distribution of data (likelihood) to yield the posterior distribution.The two main types of priors are non-informative/relatively non-informative and informative priors. A non-informative prior suggests ignorance or insufficient information about the unknown parameters of the model to help in drawing posterior inferences, while informative prior reviews what you know about parameters before the data is obtained. It comprises non-data information about the parameter of the study.

4.4.1.1 Performance of Relatively Non-informative Prior on Posterior Estimates

Table 4.21: Posterior means and Numerical standard errors of the second stage hierarchical relatively non-informative prior: when N=20, T=5

Posterior Estimates		
	Posterior Means	Numerical Standard Errors
γ	$\gamma \sim N(0,25)$, $h=0.04$	
$\delta \sim B(0,1)$	0.14828	0.00065
$\beta_0 \sim N(0,0.25)$	0.87469	0.00251
$\beta_1(2)$	1.54003	0.00233
$\beta_2(3)$	2.23406	0.00267
	$\gamma \sim N(0,30)$, $h=0.03$	
$\delta \sim B(0,1)$	0.14817	0.00059
$\beta_0 \sim N(0,0.25)$	0.87438	0.00229
$\beta_1(2)$	1.54043	0.00213
$\beta_2(3)$	2.23449	0.00244
	$\gamma \sim N(0,50)$, $h=0.02$	
$\delta \sim B(0,1)$	0.14793	0.00046
$\beta_0 \sim N(0,0.25)$	0.87368	0.00177
$\beta_1(2)$	1.54129	0.00165
$\beta_2(3)$	2.23547	0.00188
	$\gamma \sim N(0,70)$, $h=0.01$	
$\delta \sim B(0,1)$	0.14734	0.00038
$\beta_0 \sim N(0,0.25)$	0.86966	0.00153
$\beta_1(2)$	1.54315	0.00139
$\beta_2(3)$	2.23767	0.00157

Note: This Table presents the second stage of hierarchical prior

Table 4.22: Posterior means and Posterior standard deviations of the first stage hierarchical relatively non-informative prior: when N=20 and T=5, $\gamma \sim N(0, 25)$, *h=0.04*

γ	Posterior Means				Posterior Standard Deviations			
Ind	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.14052	0.88116	1.48663	2.35474	0.00351	0.00647	0.07682	0.11304
2	0.09892	0.95129	1.76378	2.31406	0.04509	0.07655	0.20037	0.07231
3	0.12482	0.96279	1.63426	2.27781	0.01926	0.08806	0.07085	0.03616
4	0.15130	0.90686	1.59088	2.16731	0.00737	0.03211	0.02733	0.07445
5	0.12067	0.85697	1.62664	2.29567	0.02346	0.01785	0.06325	0.05394
6	0.20268	0.80078	1.49368	1.99736	0.05872	0.07390	0.06980	0.24449
7	0.11991	0.93816	1.57039	2.34961	0.02411	0.06347	0.00695	0.10794
8	0.14560	0.78486	1.49223	2.32289	0.00165	0.08996	0.07137	0.08122
9	0.12361	0.94866	1.57771	2.29193	0.02044	0.07892	0.01422	0.05026
10	0.16023	0.74768	1.50081	2.28037	0.01623	0.12711	0.06264	0.03862
11	0.16049	0.94168	1.47768	2.16152	0.01645	0.06691	0.08572	0.08021
12	0.17687	0.72698	1.46649	2.14393	0.03291	0.14781	0.09692	0.09782
13	0.11798	1.02155	1.72106	2.24232	0.02601	0.14672	0.15761	0.00067
14	0.12790	0.91651	1.58667	2.31146	0.01611	0.04177	0.02322	0.06970
15	0.15460	0.94561	1.55673	2.17110	0.01061	0.07082	0.00682	0.07060
16	0.18078	0.78138	1.49518	2.13104	0.03671	0.09344	0.06833	0.11071
17	0.14147	0.72262	1.63044	2.22624	0.00252	0.15211	0.06693	0.01551
18	0.13236	0.98673	1.51299	2.32263	0.01172	0.11194	0.05050	0.10091
19	0.18804	0.73610	1.46437	2.10529	0.04400	0.13862	0.09911	0.13643
20	0.11159	0.93709	1.62096	2.34747	0.03244	0.06233	0.05744	0.10572

This Table presents the first stage of hierarchical prior $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$

Table 4.23: Posterior means and Posterior standard deviations of the first stage hierarchical relatively non-informative prior: When N=20, T=5, $\gamma \sim N(0, 30)$, *h=0.03*

γ	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.13515	0.74574	1.65266	2.25992	0.01015	0.11797	0.10918	0.00155
2	0.12988	0.98236	1.56066	2.35729	0.01549	0.11867	0.01711	0.09898
3	0.15873	0.85927	1.55589	2.16784	0.01348	0.00458	0.01238	0.09057
4	0.18393	0.89867	1.42942	2.13133	0.03870	0.03490	0.11415	0.12709
5	0.13931	0.92575	1.49919	2.35728	0.00599	0.06201	0.04434	0.09890
6	0.14315	0.81746	1.51804	2.21531	0.00219	0.04639	0.02559	0.04310
7	0.12321	0.94967	1.48616	2.41259	0.02210	0.08590	0.05741	0.15424
8	0.13122	0.88272	1.55729	2.24916	0.01404	0.01894	0.01376	0.00925
9	0.13431	0.88272	1.55729	2.24916	0.01096	0.01844	0.01286	0.06198
10	0.15444	0.83841	1.60232	2.17335	0.00924	0.02538	0.05887	0.08509
11	0.15158	0.78690	1.52948	2.26987	0.00633	0.07688	0.01414	0.01158
12	0.16991	0.82857	1.47315	2.22646	0.02468	0.03510	0.07049	0.03207
13	0.12529	0.87823	1.61488	2.34047	0.01998	0.01458	0.07136	0.08216
14	0.14879	0.80068	1.47597	2.25101	0.03510	0.06306	0.06766	0.00743
15	0.13657	0.92966	1.54692	2.35194	0.00876	0.06598	0.00347	0.09360
16	0.12919	0.76727	1.61961	2.25065	0.01618	0.09644	0.07614	0.00775
17	0.16401	0.92604	1.48861	2.21805	0.01877	0.06237	0.05496	0.04035
18	0.14527	0.80211	1.62246	2.23760	0.00007	0.06166	0.07895	0.02076
19	0.13475	0.91222	1.53457	2.25869	0.01050	0.04859	0.00894	0.00038
20	0.16656	0.89734	1.54741	2.11841	0.02138	0.03366	0.00390	0.13994

This Table presents the first stage of hierarchical prior $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$

Discussion of Results (Relatively Non-informative Prior on Posterior Estimates)

A research models used in classical techniques of a dynamic panel data model analysis can also be used in Bayesian Estimation approach. One of the qualities of a Bayesian statistics is that prior information can be included in the analysis. A non-informative prior, which is a distribution with a very high variance, does not impose strong preconditions on the parameter and the posterior estimate as a result is almost completely determined by the data. Table 4.21 presents the posterior estimates of second stage hierarchical relatively non-informative prior for $\mu_\gamma(\delta, \beta_0, \beta_1, \beta_2)$. The results show that as the precision values decrease, the posterior means of each parameter are getting closer to the initial values while their numerical standard error are reducing. Tables 4.22-4.23 reveal the posterior estimates of first stage hierarchical relatively non-informative prior. The obtained results reflect the individual contribution to the study after the variation has been removed. The results establish that the effect of the prior information depends greatly on its precision and on the variance of the variables while the structure of data has little or no impact on the posterior distribution. Hence, if the prior variance selected is high, it means researcher is very uncertain about what likely values of parameters are. As a result, the prior precision will be small and little weight will be attached to the parameter prior mean. The posterior mean attaches weight proportional to the precision of prior mean that is, the inverse of its variance.

4.4.1.2 Performance of Informative Prior on Posterior Estimates

Table 4.24: Posterior means and Numerical standard errors of the second stage hierarchical informative prior: when N=20, T=5

Posterior Estimate		
γ	Posterior means	Numerical Standard Errors
	$\gamma \sim N(0.5, 0.04)$, *h=25*	
$\delta \sim B(0,1)$	0.14779	0.00064
$\beta_0 \sim N(0, 0.25)$	0.86685	0.00257
$\beta_1(2)$	1.54183	0.00231
$\beta_2(3)$	2.23584	0.00263
	$\gamma \sim N(0.5, 0.03)$, *h=30*	
$\delta \sim B(0,1)$	0.14841	0.00060
$\beta_0 \sim N(0, 0.25)$	0.87298	0.00229
$\beta_1(2)$	1.53992	0.00213
$\beta_2(3)$	2.23363	0.00243
	$\gamma \sim N(0.5, 0.02)$, *h=50*	
$\delta \sim B(0,1)$	0.14752	0.00045
$\beta_0 \sim N(0, 0.25)$	0.86848	0.00182
$\beta_1(2)$	1.54270	0.00164
$\beta_2(3)$	2.23696	0.00186
	$\gamma \sim N(0.5, 0.01)$, *h=70*	
$\delta \sim B(0,1)$	0.14744	0.00039
$\beta_0 \sim N(0, 0.25)$	0.86905	0.00154
$\beta_1(2)$	1.54293	0.00139
$\beta_2(3)$	2.23729	0.00157

Note: This Table presents the second stage of hierarchical prior $\mu_\gamma / y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$

Table 4.25: Posterior means and Posterior standard deviations of the first stage hierarchical informative prior: When N=20, Informative prior $\gamma \sim N(0, 0.04)$, h=25, $\delta \sim B(0,1)$ $\beta_0 \sim N(0, 0.25)$

γ	Posterior Means				Posterior Standard Deviations			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.14039	0.79772	1.65594	2.19591	0.00770	0.05790	0.10356	0.02105
2	0.16700	0.70915	1.53602	2.15032	0.01896	0.14668	0.01647	0.06669
3	0.17317	0.74349	1.55046	2.0911	0.02515	0.11223	0.00197	0.12578
4	0.14813	0.96022	1.55842	2.15422	0.00005	0.10455	0.00603	0.06278
5	0.13394	0.89659	1.51130	2.28612	0.01424	0.04094	0.04114	0.06932
6	0.15253	0.76677	1.58559	2.19311	0.00445	0.08894	0.03324	0.02396
7	0.13267	0.83395	1.65969	2.24899	0.01540	0.02186	0.10735	0.03205
8	0.15959	0.82178	1.50733	2.21471	0.0115	0.03397	0.04516	0.00224
9	0.15146	0.83202	1.47379	2.25119	0.00345	0.02378	0.07869	0.03436
10	0.12195	0.83793	1.57512	2.29447	0.02627	0.01772	0.02274	0.07762
11	0.17515	0.89285	1.44786	2.16725	0.02705	0.03721	0.01050	0.04966
12	0.13996	0.78210	1.58815	2.17778	0.00811	0.07220	0.03575	0.03917
13	0.10829	0.81824	1.74515	2.27164	0.03980	0.03745	0.19280	0.05475
14	0.15423	0.91803	1.45113	2.29634	0.00614	0.06232	0.10132	0.07945
15	0.18208	0.90709	1.39879	2.17223	0.03989	0.05145	0.15360	0.04482
16	0.13300	0.92002	1.59531	2.28552	0.01511	0.06432	0.04293	0.06867
17	0.16007	0.90573	1.52946	2.14572	0.01207	0.05007	0.02291	0.07121
18	0.15094	0.95643	1.51762	2.22404	0.00286	0.10072	0.03483	0.00711
19	0.12717	0.91517	1.61197	2.29941	0.02093	0.05957	0.05950	0.08246
20	0.15026	0.89749	1.54874	2.21815	0.00221	0.04178	0.00376	0.00123

This Table presents the first stage of hierarchical prior $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$

Table 4.26: Posterior means and Posterior standard deviations of the first stage hierarchical relatively non-informative prior: When N=20, T=5, Informative prior $\gamma \sim N(0, 0.03), h{=}30$, $\delta \sim B(0,1)$ $\beta_0 \sim \text{N}(0, 0.25)$

γ	Posterior Means				Posterior Standard Deviations			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.14087	0.80456	1.64635	2.20009	0.00707	0.05296	0.09464	0.01904
2	0.16516	0.72357	1.53688	2.15854	0.01727	0.13390	0.01485	0.06059
3	0.17079	0.75484	1.55009	2.10447	0.02296	0.10275	0.00168	0.11468
4	0.14806	0.95291	1.55726	2.16167	0.00008	0.09536	0.00558	0.05743
5	0.13504	0.89498	1.51418	2.28227	0.01293	0.03748	0.03755	0.06310
6	0.15196	0.77625	1.58211	2.19748	0.00409	0.08139	0.03047	0.02162
7	0.13384	0.83774	1.64974	2.24849	0.01414	0.01980	0.09790	0.02948
8	0.15844	0.82655	1.51062	2.21712	0.01048	0.03106	0.04116	0.00196
9	0.15101	0.83595	1.47996	2.25039	0.00316	0.02166	0.07174	0.03134
10	0.12407	0.84142	1.57247	2.28997	0.02394	0.01615	0.02078	0.07095
11	0.17266	0.89139	1.45631	2.17362	0.02471	0.03393	0.09543	0.04542
12	0.14049	0.79151	1.58446	2.18351	0.00750	0.06604	0.03271	0.03552
13	0.11156	0.82345	1.72776	2.26925	0.03639	0.03419	0.17605	0.05023
14	0.15359	0.91457	1.45922	2.29146	0.00565	0.05705	0.09254	0.07239
15	0.17901	0.90438	1.41150	2.17812	0.03114	0.04686	0.14036	0.04092
16	0.13418	0.91639	1.59089	2.28167	0.01370	0.05882	0.03913	0.06252
17	0.15889	0.90313	1.53084	2.15399	0.01092	0.04561	0.02097	0.06514
18	0.15056	0.94955	1.51997	2.22542	0.00262	0.09207	0.05434	0.00634
19	0.12885	0.91198	1.60609	2.29437	0.01919	0.05444	0.05433	0.07524
20	0.14994	0.89572	1.54840	2.22015	0.00190	0.03816	0.00346	0.00117

This Table presents the first stage of hierarchical prior $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$

Discussion of Results (Informative Prior on Posterior Estimates)

Informative prior is appropriate when there is adequate prior information on the scale and shape of the distribution of a model parameter. The suitable information about the parameters is incorporated into the prior distribution. The variations within variables are considered and specifically interesting to discover appropriate prior information, since data only cannot provide all information needed for analysis. Therefore, Table 4.24 presents the posterior estimates of second stage hierarchical informative prior for $\mu_{\gamma}(\delta,\beta_0,\beta_1,\beta_2)$. This table produced good estimates of posterior mean as the error variance decreases and error precision increases, the results approach the initial values steadily while the numerical standard error decreases as the error precision increases indicate an approximation error which is small relative to posterior standard deviations of all parameters while Tables 4.25 and 4.26 reflect the individual contribution of explanatory variables towards the dependent variable. The tables show a substantial influence of the prior information.The more precise the prior, the bigger its influence.

It is observed that as h changes in values, the posterior mean of both the relatively noninformative prior and informative prior approached the true value at every stage of the error precision which are closely identical to the pre-set β_1 and β_2 values.

It is also important to note that posterior estimates of informative prior are more appropriate in terms of minimum numerical standard error and closer posterior mean to the initial values compared to estimates of relatively non-informative prior. This is an indication that the data and information about the parameters are very sensitive to the posterior distribution. Hence, Bayesian method is found relevant when sample sizes are small or large relative to the number of parameters.

Therefore, Hierarchical Bayesian methods combine data and prior information is of great technique to the study of unobserved individual heterogeneity dynamic panel data model.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Introduction

Estimation of parameters of a dynamic heterogeneous panel data model is an essential task in economics. It often requires appropriate estimators for the small, moderate and large dimensions of N (the number of units) and T (the number of time periods) with slope heterogeneity. When regression coefficients vary across the units, pooling of data in a dynamic model gives inconsistent and misleading estimates of the slope coefficients. The problem of heterogeneity in the dynamic panel data model may be as a result of omitted information in the regression equation and, in many study, it is more realistic to model the variance of the random coefficients which differ across the units. Ignoring heterogeneity when it is present yields efficient estimates of the regression coefficients but these estimates will not be consistent and their standard errors will be biased.

Bayesian method becomes essential when researcher wishes to make statistical inference on the slope coefficients distribution of when sufficient or insufficient knowledge about parameters of the model is very important.

5.2 Summary of the findings

In this study a more general framework is provided where the coefficients of the lagged dependent variable, intercept and slopes are functions of a set of independent variables (random coefficient model) and are randomly drawn from a certain distribution. The presence of lagged dependent variable among the explanatory variables raises a problem of endogeneity since they are function of the individual effects.

The hierarchical Bayesian estimator was derived to eliminate the problem of heterogeneity in the dynamic panel data model which improved the non-consistent estimates of the parameters of the classical approach.

The findings of the results with respect to the two stages of hierarchical Bayesian estimator of their performances in terms of posterior mean, posterior standard deviation and numerical standard error which were displayed in the previous tables and figures are as follows:

(i) The results of the second stage hierarchical Bayesian estimates are presented in Tables 4.1, 4.5 and 4.9. It was observed that as N increases the posterior mean of the slope coefficients are closer to the initial values which are set to be 2 and 3 when the data was generated. The numerical standard errors of the parameters consistently decreased as N increased for all values of T considered. The posterior estimates for δ *and* β_0 are obtained within the stipulated distribution range, indicating that the estimator provide good estimates for all dimension of N and T.

(ii) Tables 4.2-4.4, 4.6-4.8 and 4.10-4.12 presented results for the first stage hierarchical Bayesian estimates. These results revealed the impact of each individual on the dependent variable indicating that the MCMC approach effectively handled variation among parameters across individual. The numerical standard error indicates an approximation error obtained in second stage hierarchical prior small relative to posterior standard deviation in first stage hierarchical prior error of all parameters.

(iii) Figures 4.1(a-c), 4.5(a-c), 4.9(a-c), 4.13, 4.15, 4.17 and 4.19 displayed histogram graphs for the second stage hierarchical Bayesian estimates. They presented information about parameters in the model using the simulated datasets while the figures look similar to one another, as the values of N and T change, indicating that the hierarchical priors are providing basically closed estimates of each parameter.

(iv) Figures 4.2-4.4, 4.6-4.8, 4.10-4.12, 4.14, 4.16, 4.18 and 4.20 displayed histogram graphs for the first stage hierarchical Bayesian estimates. The graph displays the individual pattern over time period which exhibit the true picture of each parameter across individuals, indicating the real influence of independent variables over dependent variable. An examination of all these figures shows that regression coefficient models are doing a very good job at picking out the variation in each parameter.

(v) Our finding indicated that $N > \mathrm{T}$ option in experiment III produced the least Numerical Standard Error (NSE), hence outperformed the other two experiments.

(vi) The obtained error variance-covariance matrix produced a constant error variance (V_γ) for all the parameters across the units through the generalization of Gamma distribution (Wishart) indicate homogeneity property within the parameters.

(vii) The potential achievement of the estimation results are facilitated by suitable prior information. The estimated parameters with changes in h values were closely identical to the pre-set β_1 and β_2 values. Thus, indicating the sensitivity of prior information on the posterior estimates.

5.3 Conclusions

This study uses a dynamic panel model to examine how unobserved individual heterogeneity affects parameters of inference. It is observed that not accounting for the heterogeneity yields non-consistent estimates of the mean of dynamic panel data model, even with large N and T. Therefore, a great deal of interest was placed on hierarchical Bayesian estimation of unobserved individual heterogeneity of dynamic panel data model, in order to improve on a static panel data model. The method allows for unit-specific coefficients to be different across observations and imposing a stability condition for individual autoregressive coefficient drawn from a beta distribution (0, 1). The theoretical findings are accompanied by extensive Markov Chain Monte Carlo (MCMC) experiments. The examination of all the figures and tables indicate that the hierarchical Bayesian method effectively handled the complicated pattern exhibited by the data as the dimensions of N and T change.

The results show that the parameter model with insufficient information (relatively non informative prior) and sufficient information (informative prior) produce consistent parameter estimates.

The hierarchical Bayesian estimator facilitated by suitable prior information solved the problem of heterogeneity in the dynamic panel data model. Therefore, the estimator will find useful applications in panel data economic models.

5.4 Research Contributions to Knowledge

This study contributes to the existing literature in the following distinctive ways:

(i) It derived a hierarchical Bayesian estimator to estimate random coefficients of a dynamic panel data model as against the static panel data model proposed by Koop (2003).

(ii) It imposed a stationarity assumption for each unit's process by assuming that the unit- specific autoregressive coefficient (δ_i) of a lagged dependent variable is drawn from a beta distribution whose support is (0, 1) with the stationarity condition $|\delta_i|<1$ as against the logitnormal distribution (-1,1) imposed on δ_i proposed by Zhang and Small (2006)

(iii) It developed a process to check for the sensitivity of prior information via relative non-informative and informative prior on the posterior estimate of the dynamic panel data model.

(iv) Different dimension of N and T (N<T, N=T, N>T) were examined which no single study has been able to compare.

(v) An R code was written for Gibbs Sampling algorithm to obtain a consistent estimate of dynamic panel data model.

5.5 Limitations of the study

The study was with several limitations. Below are few of them:

(i) One of the difficulties experienced was that of R code unable to produce results on time and unreliable computer components (such as computer charger, battery). Writing of program that will successfully yield a reasonable result becomes a big challenge.

(ii) Different stages involved in the hierarchical Bayesian estimates distort the free flow of the simulated data and also the systematically manner of injecting second stage of hierarchical prior into first stage of hierarchical prior was not found easy.

5.6 Recommendations for further research

Based on the results of the analysis carried out, the outcomes of our findings and conclusions give out the following recommendations to researchers in Bayesian econometrics:

(i) The problem of heterogeneity should not be over looked in the estimation of dynamic panel data model to obtain consistent estimates of the parameters.

(ii) The use of hierarchical prior in Bayesian estimation should be in practice especially when parameters space is of high dimensions.

(iii) Coefficient of a lagged dependent variable should be generated from a distribution whose support is (0, 1) in order to establish its stationary condition.

(iv) Appropriate prior information should be chosen in the estimation of parameter model for the Bayesian approach to be efficient.

(v) Researchers are encouraged to account for individual specific effects capable of giving the true picture of the relationship between dependent variable and independent variables.

5.7 Suggestions for Further Research

Thisstudy has obviously obtained a consistent parameter estimates through a hierarchical Bayesian estimate of a dynamic panel data model.

(i) The study focuses on model whose random coefficients differ across the units. Therefore, further study can considera dynamic panel data model of the random coefficients which differ across both the units and the times.

(ii) This study can also be extended to dynamic panel data of non-linear model.

(iii) We use a Gibbs sampling algorithm to generate the hierarchical Bayesian estimator; further research can consider a Metropolis-within-Gibbs-Sampling algorithm to generate its Bayes estimates.

(iv) It is suggested to compare the classical approach of a dynamic panel data model with Bayesian approach of purely noninformative prior in order to establish their equivalent according to Koop (2003).

(v) Different prior distributions can also be used to investigate the prior sensitivity on posterior estimates.

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APPENDIX I

R-Written Code For Dimension (N, T)

```
rm(list = ls())
unlink("R.data")
getwd()
setwd("C:/Users/oluwalovesme/Desktop/Akinlade")
#--------------------------------------------------
norm_rnd = function(sig){
  # if nargin ~= 1 error('Wrong # of arguments to norm_rnd')
  h = chol(sig)
  nrow = dim(sig)[1]
  x = rnorm(nrow,1)
  y = t(h)*x
}
######################################################################
####
gamm_rnd = function(nrow, ncol, m, k){
  # if(nargin != 4){  error('Wrong # of arguments to gamm_rnd')}
  gb = matrix(0, nrow=nrow, ncol=ncol)
  if(m <= 1){
    # Use RGS algorithm by Best, p. 426
    c = 1/m
    t = 0.07 + 0.75 * sqrt(1-m)
    b = 1 + exp(-t) * m/t
    for(i1 in 1:nrow){
      for(i2 in 1:ncol){
        accept = 0
        while(accept == 0){
          u = runif(1); w = runif(1); v = b * u;
          if(v<=1){
            x = t * (v^c);
            accept = ((w <= ((2-x)/(2+x))) | (w <= exp(-x)))
          } else{
            x=-log(c*t*(b-v))
```

```
        y=x/t
        accept = (((w*(m+y-m*y))<=1) | (w<=(y^(m-1))));

      gb[i1,i2] = x
    }
  }else{
    # Use Best's rejection algorithm XG, p. 410
    b = m - 1
    c = 3 * m - 0.75
    for(i1 in 1:nrow){
      for(i2 in 1:ncol){
        accept = 0
        while(accept == 0){
          u = runif(1);  v = runif(1);
          w = u * (1 - u);  y = sqrt(c/w)*(u-0.5);
          x = b + y
          if(x >= 0){
            z = 64 * (w^3) * v * v;
            accept = (z<=(1-2*y*y/x)) | (log(z)<=(2*(b*log(x/b)-y)))
          }
        }
        gb[i1,i2] = x
      }
    }
    gb = gb/k
  }
}
######################################################################
#####

######################################################################
#####
####################      logwish_pdf function      ####################
```

```
########################################################################
#####
# PURPOSE: log of pdf of the Wishart(A,a) distribution evaluated at Z
# defined as in Poirier (1995) page 136
# ---------------------------------------------------------------
logwish_pdf = function(Z, A, omega){
  M = nrow(Z)
  lintcon = .5*omega*M*log(2) + .25*M*(M-1)*log(pi)
  for(i in 1:M){
    lintcon = lintcon + log(gamma(.5*(omega + 1-i)))
  }
  pdf = -lintcon + .5*(omega-M-1)*log(det(Z)) - .5*omega*log(det(A))
  - .5*sum(diag(solve(A)*Z))
}
########################################################################
#####
#################     End of logwish_pdf function      ##################
########################################################################
#####
wish_rnd = function(sigma,v){
  n = dim(sigma)[1]
  k = dim(sigma)[2]
  # if(n != k){error('wish_rnd: requires a square matrix')} else
  # if(n < k){warning('wish_rnd: n must be >= k+1 for a finite distribution')}
  tp = chol(sigma)
  t = tp[1]
  p = tp[2]
  # if(p < 0){error('wish_rnd: matrix must be a positive definite')}
  y = t(t) * rnorm(n,v)
  w = y %*% t(y)
}
########################################################################
#####
#################     End of wish_rnd function      ##################
```

```
########################################################################
#####
momentg <- function(draws){
 results = dim(draws);  ndraw = as.numeric(results[1]);
 nvar = as.numeric(results[2]);
 results_pmean = numeric(nvar);  results_pstd = numeric(nvar)
 results_rne = numeric(nvar);  results_rne1 = numeric(nvar)
 results_rne2 = numeric(nvar);  results_rne3 = numeric(nvar)
 results_nse = numeric(nvar);  results_nse1 = numeric(nvar)
 results_nse2 = numeric(nvar);  results_nse3 = numeric(nvar)
 # results.meth = "momentg"
 NG = 100;
 ntaper = c(4, 8, 15);  ns = floor(ndraw/NG); nuse = ns*NG

 eg = numeric(nvar); varg = numeric(nvar)
 ann=0; add=0; and=0; adn=0
 rnn = numeric(NG);  rdd = numeric(NG);  rnd = numeric(NG);
 rdn = numeric(NG);
 eg = numeric(nvar); varg = numeric(nvar); sdnum = numeric(nvar);
 varnum = numeric(nvar)
 for(j in 1:nvar){     # loop over all variables
  cnt = 0
  cn = rep(0,NG);  cd = rep(0,NG); cdn = rep(0,NG); cdd = rep(0,NG);
  cnn = rep(0,NG); cvar = rep(0,NG)
  td = 0; tn = 0; tdd = 0; tnn = 0; tdn = 0; tvar = 0
  for(i in 1:NG){     # form sufficiency statistics needed below
   gd = 0; gn = 0; gdd = 0; gdn = 0; gnn = 0; gvar = 0
   for(k in 1:ns){
    cnt = cnt + 1; g = draws[cnt,j];  ad=1; an=ad*g;
    gd = gd + ad; gn = gn + an; gdn = gdn + (ad*an);
    gdd = gdd + (ad*ad);  gnn = gnn + (an*an);
    gvar = gvar + (an*g);
   }
   td = td + gd; tn = tn + gn; tdn = tdn + gdn; tdd = tdd + gdd;
```

```
  tnn = tnn + gnn; tvar = tvar + gvar;
  cn[i] = gn/ns; cd[i] = gd/ns; cdn[i] = gdn/ns; cdd[i] = gdd/ns;
  cnn[i] = gnn/ns; cvar[i] = gvar/ns;
}
eg = tn/td
varg = (tvar/td) - (eg^2)
sdg = -1
if(varg > 0){
  sdg = sqrt(varg)
} else sdg = sdg
# save posterior means and std deviations to results structure
results_pmean[j] = eg
results_pstd[j] = sdg

# numerical standard error assuming no serial correlation
varnum = (tnn - 2*eg*tdn + tdd*eg^2)/(td^2);
sdnum = -1;
if(varnum > 0){
  sdnum = sqrt(varnum)
}
# save to results structure
results_nse[j] = sdnum;  results_rne[j] = varg/(nuse*varnum);

# get autocovariance of grouped means
barn = tn/nuse; bard = td/nuse;
for(i in 1:NG){
  cn[i] = cn[i] - barn;  cd[i] = cd[i] - bard;
}
Ng = NG-1
for(l in 0:Ng){
  h = l + 1 # To avoid complexities
  for(i in h:NG){
    ann = ann + cn[i]*cn[i-l];   add = add + cd[i]*cd[i-l];
    and = and + cn[i]*cd[i-l];   adn = adn + cd[i]*cd[i-l];
```

```
      }
      # index 0 not allowed, lag+1 stands for lag
      rnn[h] = ann/NG;  rdd[h] = add/NG;   rnd[h] = and/NG;
      rdn[h] = adn/NG;
    }
    # numerical standard error with tapered autocovariance functions
    for(mm in 1:3){
      m = ntaper[mm]; am = m;
      snn = rnn[1]; sdd = rdd[1]; snd = rnd[1];
      for(lag in 1:(m-1)){
        att = 1 - lag/am;          snn = snn + 2*att*rnn[h];
        sdd = sdd + 2*att*rdd[h];  snd = snd + att*(rnd[h] + rnd[h]);
      }
      varnum = ns*nuse*(snn - 2*eg*snd + sdd*eg^2)/(td^2);
      sdnum = -1;
      if(varnum>0){
        sdnum = sqrt(varnum)
      }
      # save results in structure
      if(mm == 1){
        results_nse1[j] = sdnum;  results_rne1[j] = varg/(nuse*varnum);
      } else
        if(mm == 2){
          results_nse2[j] = sdnum; results_rne2[j] = varg/(nuse*varnum);
        } else
          if(mm == 3){
            results_nse3[j] = sdnum; results_rne3[j] = varg/(nuse*varnum);
          }
    }
  }
  rsltmntg = cbind(results_pmean, results_pstd, results_nse, results_nse1,
                   results_nse2, results_nse3, results_rne, results_rne1,
                   results_rne2, results_rne3)
}
```

```
########################################################################
#####
###############                  END    OF    MOMENTG    FUNCTION
#########################
########################################################################
#####

#Generate artificial data on the explanatory variable
# simulation data
n=50
t=50
tn=t*n
library("dyn")
set.seed(134)
tz    <-    zoo(cbind(Y    =    0,lagpara=rbeta(2501,0,1),xo=rnorm(2501,0,0.25),
x1=runif(2501),x2=runif(2501), e=rnorm(2501)))
# simulate values
for(i in 2:2501) {
  tz$Y[i] <- with(as.data.frame(tz),
          lagpara*Y[i-1]+xo+2*x1[i]+3*x2[i] + e[i])
}
y=tz$Y[2:2501]
lagy=tz$Y[1:2500]
xo=tz$xo[1:2500]
x1=tz$x1[1:2500]
x2=tz$x2[1:2500]
x=cbind(lagy,xo,x1,x2)
xpx=t(x)%*%(x)     # x'x
xpxinv=solve(xpx)   # (x'x)^-1
xpy=t(x)%*%(y)     #  x'y
bols=(xpxinv)%*%(xpy)
# posterior distribution
library(bayesm)
n=50
```

```
t=50
tn=t*n
h02=25
s02=1/h02
k=4
rho = 2
# Define the prior hyperparameters
sigma=(1/h02)
xpx=t(x)%*%(x)      # x'x
xpxinv=solve(xpx)    # (x'x)^-1
xpy=t(x)%*%(y)      #  x'y
bols=(xpxinv)%*%(xpy)   # B=(x'x)^-1*(x'y)
s2 = t(y - x%*%bols) %*% (y - x%*%bols)/(tn-k)[1]

# Hyperparameters for independent Normal-Gamma prior
v0=1
v1= v0 + tn
smallv=2
sigmabeta=solve(diag(4));
priorv=diag(4);
vbar=diag(4)
mu_beta=matrix(c(0,0,0,0),byrow=T )
smallv_bar=n+smallv
bigv_bar=(bols-(mu_beta))%*%t(bols-(mu_beta))+vbar

xsquare = t(x) %*% x
v0s02 = v0 * h02
vrho = rho + n

s=solve(smallv_bar*bigv_bar)
priorv=rWishart(1,smallv_bar,s)
priorv=matrix((priorv),ncol=4,nrow=4)
deg=n*priorv+sigmabeta
# choose a starting value for h
```

```
hdraw = 1/s2
# If imlike==1 then calculate marginal likelihood, if not then no marglike
imlike = 1
if (imlike==1){
  bchib = bols
  hchib = 1/s02

  # log prior evaluated at this point
  b0 = matrix(0,nrow = k,ncol = 1)
  logprior = -.5*v0*log(2*h02/v0) - gamma(log(.5*v0)) + .5*(v0-2)*log(hchib) -
.5*v0*hchib/h02 -.5*k*log(2*pi) -.5*k*log(det(priorv))-.5*t(bchib-b0) %*% deg
%*% (bchib-b0)

  # log likelihood evaluated at the point
  loglike = -.5*tn*log(2*pi)+.5*tn*log(hchib)-.5*hchib*t(y - x%*%bchib) %*% (y
- x%*%bchib)
}

loglike

# Use Chib (1995) method for marginal likelihood calculation
# this requires point to evaluate all at --- try ols results or use post means

# Specify the number of replications
# number of burnin replications
s0 = 1000
# number of retained replications
s1 = 10000
sk = s0+s1

# store all draws in the following matrices
# initialize them here
b_ = matrix(nrow = sk, ncol = k)
h_ = matrix(nrow = sk, ncol = 1)
```

```
logpost2 = 0;
# Now start Gibbs loop
# beta conditional on h is Normal
# h conditional on beta is Normal
for (i in 1:sk){
  # draw from beta conditional on h
  library(MASS)
  post_sigmabeta=solve(deg)
  f=((sigmabeta)%*%(mu_beta))
  post_meanbeta=post_sigmabeta%*%((priorv%*%bols)+f)
  beta_mu=mvrnorm(1,post_meanbeta,post_sigmabeta)
  library(MASS)
  postv=solve(priorv+(drop(h02)*xpx))
  postbeta=postv%*%(h02*xpy+(priorv%*%beta_mu))
  mubeta=mvrnorm(1,postbeta,postv)
  mubeta=as.matrix(mubeta)
  mubeta # first stage

  # posterior conditional for the error precision
  # draw from h conditional on beta
  s12 = (t(y - x %*% mubeta) %*% (y - x %*% mubeta) + v0s02)/v1
  precision = rgamma(1, .20*v1, s12)

  if (i>s0){
    # after discarding burnin, store all draws
    b_[i,] = mubeta
    h_[i,] = precision

    if (imlike==1){
      # log posterior for betaevaluated at point -- use for marg like
      # see Chib (1995, JASA) pp. 1315 for justification
      logpost = -.5*k*log(2*pi) -.5*k*log(det(post_sigmabeta)) - .5*t(bchib-f) %*%
deg %*% (bchib-f)
```

```
if (imlike ==1){
  logpost2 = logpost2/s1
  # we need p(beta,h|y) evaluated as point
  # In loop we calculated p(beta|y) now need p(h|y,beta) to complete
  s12 = (t(y - x %*% bchib) %*% (y - x %*% bchib) + smallv_bar)/v1
  logpost1 = -.5*v1*log(2/(v1*s12)) - gamma(log(.5*v1)) + .5*(v1-2)*log(hchib) -
.5*v1*hchib*s12
  logmlike = loglike + logprior - logpost2 - logpost1;
}
alldraws = cbind(b_, h_)
alldraws7d = alldraws[101:sk,]
result7d = momentg(alldraws7d)
# Posterior results based on Informative Prior
# number of burnin replications
# Trace plot
plot(h_,type="l")
plot(b_[,1],type="l")
plot(b_[,2],type="l",col="red")
plot(b_[,3],type="l")
plot(b_[,4],type="l",col="blue")

#Histogram
f<-b_[,1]
k<-b_[,2]
l<-b_[,3]
m<-b_[,4]

fa<-(f[101:1100])
lagymu<-hist(fa, main="Histogram of lag y")
xfit<-seq(min(fa),max(fa),length=10)
yfit<-dnorm(xfit,mean=mean(fa),sd=sd(fa))
yfit <- yfit*diff((lagymu)$mids[1:2])*length(fa)
lines(xfit, yfit, col="blue", lwd=2)
plot(density(fa),main="density of lag y")
```

```
ka<-(k[101:1100])
beta0mu<-hist(ka, main="Histogram of beta 0")
xfit<-seq(min(ka),max(ka),length=10)
yfit<-dnorm(xfit,mean=mean(ka),sd=sd(ka))
yfit <- yfit*diff((beta0mu)$mids[1:2])*length(ka)
lines(xfit, yfit, col="red", lwd=2)
plot(density(ka),main="density of beta 0")

la<-(l[101:1100])
beta1mu<-hist(la, main="Histogram of beta 1")
xfit<-seq(min(la),max(la),length=10)
yfit<-dnorm(xfit,mean=mean(la),sd=sd(la))
yfit <- yfit*diff((beta1mu)$mids[1:2])*length(la)
lines(xfit, yfit, col="red", lwd=2)
plot(density(la),main="density of beta 1")
ma<-(m[101:1100])
beta2mu<-hist(ma, main="Histogram of beta 2")
xfit<-seq(min(ma),max(ma),length=10)
yfit<-dnorm(xfit,mean=mean(ma),sd=sd(ma))
yfit <- yfit*diff((beta2mu)$mids[1:2])*length(ma)
lines(xfit, yfit, col="blue", lwd=2)
plot(density(ma),main="density of beta 2")
#lagy
par(mfrow=c(2,3))
lagymu<-hist(fa, main="Histogram of lag y")
plot(density(fa),main="density of lag y")
plot(b_[,1],type="l")

lagymu<-hist(fa, main="Histogram of lag y")
xfit<-seq(min(fa),max(fa),length=10)
yfit<-dnorm(xfit,mean=mean(fa),sd=sd(fa))
yfit <- yfit*diff((lagymu)$mids[1:2])*length(fa)
lines(xfit, yfit, col="blue", lwd=2)
```

```
#Beta 0
par(mfrow=c(2,3))
beta0mu<-hist(ka, main="Histogram of beta 0")
plot(density(ka),main="density of beta 0")
plot(b_[,2],type="l",col="red")

beta0mu<-hist(ka, main="Histogram of beta 0")
xfit<-seq(min(ka),max(ka),length=10)
yfit<-dnorm(xfit,mean=mean(ka),sd=sd(ka))
yfit <- yfit*diff((beta0mu)$mids[1:2])*length(ka)
lines(xfit, yfit, col="red", lwd=2)


#Beta 1
par(mfrow=c(2,3))
beta1mu<-hist(la, main="Histogram of beta 1")
plot(density(la),main="density of beta 1")
plot(b_[,3],type="l")

beta1mu<-hist(la, main="Histogram of beta 1")
xfit<-seq(min(la),max(la),length=10)
yfit<-dnorm(xfit,mean=mean(la),sd=sd(la))
yfit <- yfit*diff((beta1mu)$mids[1:2])*length(la)
lines(xfit, yfit, col="red", lwd=2)

#Beta 2
par(mfrow=c(2,3))
beta2mu<-hist(ma, main="Histogram of beta 2")
plot(density(ma),main="density of beta 2")
plot(b_[,4],type="l",col="blue")

beta2mu<-hist(ma, main="Histogram of beta 2")
xfit<-seq(min(ma),max(ma),length=10)
yfit<-dnorm(xfit,mean=mean(ma),sd=sd(ma))
```

```
yfit <- yfit*diff((beta2mu)$mids[1:2])*length(ma)
lines(xfit, yfit, col="blue", lwd=2)

######################################################################
######################################################################
######################################################################
################
par(mfrow=c(2,2))
fa<-(f[101:1100])
lagymu<-hist(fa, main="Histogram of lag y")
xfit<-seq(min(fa),max(fa),length=10)
yfit<-dnorm(xfit,mean=mean(fa),sd=sd(fa))
yfit <- yfit*diff((lagymu)$mids[1:2])*length(fa)
lines(xfit, yfit, col="blue", lwd=2)
ka<-(k[101:1100])
beta0mu<-hist(ka, main="Histogram of beta 0")
xfit<-seq(min(ka),max(ka),length=10)
yfit<-dnorm(xfit,mean=mean(ka),sd=sd(ka))
yfit <- yfit*diff((beta0mu)$mids[1:2])*length(ka)
lines(xfit, yfit, col="red", lwd=2)

la<-(l[101:1100])
beta1mu<-hist(la, main="Histogram of beta 1")
xfit<-seq(min(la),max(la),length=10)
yfit<-dnorm(xfit,mean=mean(la),sd=sd(la))
yfit <- yfit*diff((beta1mu)$mids[1:2])*length(la)
lines(xfit, yfit, col="red", lwd=2)
ma<-(m[101:1100])
beta2mu<-hist(ma, main="Histogram of beta 2")
xfit<-seq(min(ma),max(ma),length=10)
yfit<-dnorm(xfit,mean=mean(ma),sd=sd(ma))
yfit <- yfit*diff((beta2mu)$mids[1:2])*length(ma)
lines(xfit, yfit, col="blue", lwd=2
s0
```

```
# number of included replications
s1
if (imlike==1){
 # Log of Marginal Likelihood
 logmlike
}
# Posterior means, std. devs and nse for parameters
# Parameters ordered as theta, gamma then error precision
# Save the posterior means for use as point at which to evaluate in Chib method
chibval7d = result7d[,1:3]
rownames(chibval7d ) = c("lagpar","beta-0", "beta-1","beta-2","precision")
chibval7d

############################stagetwo###################################
#############
# theta0 has Normal prior with mean mu_theta and variance V_theta
# --------
# Define the prior hyperparameters
# --------
# hierarchical prior for varying coefficients
# Notation used here has theta0 being mean of theta in hierarchical prior
# theta0 has Normal prior with mean mu_theta and variance V_theta
kx=4
mu_theta = matrix(0,kx,1)
V_theta = 1*diag(kx)
V_thinv = solve(V_theta)
# Notation here has Sigma being variance of theta in hierarchical prior
# Sigma-inverse has prior which is Wishart
# degree of freedom = rho, scale matrix R --- implying mean = rho*R
n=50
rho = 2
k=4
R = as.matrix(.5*diag(kx))
# for error precision use Gamma prior with mean h02 and v0=d.o.f.
```

```
v0 = 1
h02 = 25
s02 = 1/h02
# Do OLS and related results (assuming no heterogeneity) to get starting values
bols = solve(t(x) %*% x) %*% t(x) %*% y
s2 = t(y - x%*%bols)%*%(y - x %*% bols)/(tn-kx)
# choose a starting value for h
hdraw = 1/s2
# Calculate a few quantities outside the loop for later use
xsquare = t(x) %*% x
v1 = v0 + tn
v0s02 = v0 * h02
vrho = rho + n
# capital sigma inverse
sigchib = diag(kx)
# sigchib = chibval7c[(k+2):(k+kx+1), 1]
sigichib = solve(sigchib)
th0chib = bols[1:(kx),1]
gchib = bols[(kx):k,1]
hchib = 1/s2
# starting value for theta
#thetdraw = cbind(bols[1,1] * matrix(1,n),bols[2,1]*matrix(1,n),bols[3,1]*matrix(1,n))
#thet0draw = cbind(bols[1,1],bols[2,1],bols[3,1])

xsquare = t(x) %*% x
v0s02 = v0 * h02
vrho = rho + n

s=solve(smallv_bar*bigv_bar)
priorv=rWishart(1,smallv_bar,s)
priorv=matrix((priorv),ncol=4,nrow=4)
deg=n*priorv+sigmabeta
# choose a starting value for h
```

```
hdraw = 1/s2
# If imlike==1 then calculate marginal likelihood, if not then no marglike
imlike = 1
if (imlike==1){
  bchib = bols
  hchib = 1/s02
  # log prior evaluated at this point
  b0 = matrix(0,nrow = k,ncol = 1)
  logprior = -.5*v0*log(2*h02/v0) - gamma(log(.5*v0)) + .5*(v0-2)*log(hchib) -
.5*v0*hchib/h02 -.5*k*log(2*pi) -.5*k*log(det(priorv))-.5*t(bchib-b0) %*% deg
%*% (bchib-b0)

  # log likelihood evaluated at the point
  loglike = -.5*tn*log(2*pi)+.5*tn*log(hchib)-.5*hchib*t(y - x%*%bchib) %*% (y
- x%*%bchib)

loglike

# Use Chib (1995) method for marginal likelihood calculation
# this requires point to evaluate all at --- try ols results or use post means
# Specify the number of replications
# number of burnin replications
s0 = 100
# number of retained replications
s1 = 1000
sk = s0+s1
# store all draws in the following matrices
# initialize them here
h_ = matrix(nrow=sk, ncol=1)
th0_ = matrix(nrow=sk, ncol=kx)
sig_ = matrix(nrow=sk, ncol=kx^2)

# Now start Gibbs loop
# beta conditional on h is Normal
```

```
# h conditional on beta is Normal
for (i in 1:sk){
  sigterm = matrix(0, nrow=kx, ncol=kx)
  sigterm1 = solve(sigterm + rho * R)
  siginv = rWishart(1,smallv_bar,sigterm1) # Replaced vrho with 1
  sigdraw=matrix((siginv),nrow=4,ncol=4)
  sigdraw=solve(sigdraw)
  temp =matrix(sigdraw,nrow=1,ncol=(kx)^2);# # Check reshape(sigdraw, kx^2,
1)

  # draw from h conditional on other parameters
  s12 = (t(y - x %*% mubeta) %*% (y - x %*% mubeta) + v0s02)/v1
  precision = rgamma(1, .20*v1, s12);
  # Now draw theta0 (mean in hierarchical prior)conditional on other params

  # Now draw theta-i s
  s=solve(smallv_bar*bigv_bar)
  priorv=rWishart(1,smallv_bar,s)
  priorv=matrix((priorv),ncol=4,nrow=4)
  deg=n*priorv+sigmabeta
  post_sigmabeta=solve(deg)
  f=((sigmabeta)%*%(mu_beta))
  post_meanbeta=post_sigmabeta%*%((priorv%*%bols)+f)
  library(MASS)
  beta_mu=mvrnorm(1,post_meanbeta,post_sigmabeta)
  library(MASS)
  postv=solve(priorv+(drop(h02)*xpx))
  postbeta=postv%*%(h02*xpy+(priorv%*%beta_mu))
  beta=mvrnorm(1,postbeta,postv)
  beta;

  if(i>s0){
    # after discarding burnin, store all draws
    h_[i,] = precision
```

```
    th0_[i,] = beta
    sig_[i,] = temp # cbind(sig_, temp);
    if (imlike==1){
      # log posterior for betaevaluated at point -- use for marg like
      # see Chib (1995, JASA) pp. 1315 for justification
      logpost = -.5*k*log(2*pi) -.5*k*log(det(post_sigmabeta)) - .5*t(bchib-f) %*%
deg %*% (bchib-f)
      logpost2 = logpost2 + logpost
if (imlike ==1){
  logpost2 = logpost2/s1
  # we need p(beta,h|y) evaluated as point
  # In loop we calculated p(beta|y) now need p(h|y,beta) to complete
  s12 = (t(y - x %*% bchib) %*% (y - x %*% bchib) + smallv_bar)/v1
  logpost1 = -.5*v1*log(2/(v1*s12)) - gamma(log(.5*v1)) + .5*(v1-2)*log(hchib) -
.5*v1*hchib*s12
  logmlike = loglike + logprior - logpost2 - logpost1;
}
alldraws = cbind(h_,th0_,sig_)
alldraws7db = alldraws[101:sk,]
result7db = momentg(alldraws7db)

plot(h_,type="l")
plot(th0_[,1],type="l",col="yellow")
plot(th0_[,2],type="l",col="red")
plot(th0_[,3],type="l",col="green")
plot(th0_[,4],type="l",col="blue")

# Posterior results'
# number of burnin replications'
s0
# number of included replications'
s1

#if(imlike==1){
```

```
# Posterior means, std. devs and nse for parameters
# Parameters ordered as error precision, theta0, vec(sig)
A=result7db[,1:3]
rownames(A      )      =      c("precision","lagpar","beta-0",      "beta-1","beta-
2","vec1","vec2","vec3","vec4","vec5","vec6","vec7","vec8","vec9","vec10","
vec11","vec12","vec13","vec14","vec15","vec16")
thmean = matrix(0, nrow = n, ncol = kx)
thsd = matrix(0, nrow = n, ncol = kx)

# Now draw theta-i s
s=solve(smallv_bar*bigv_bar)
priorv=rWishart(1,smallv_bar,s)
priorv=matrix((priorv),ncol=4,nrow=4)
deg=n*priorv+sigmabeta
post_sigmabeta=solve(deg)
f=((sigmabeta)%*%(mu_beta))
post_meanbeta=post_sigmabeta%*%((priorv%*%bols)+f)
library(MASS)
beta_mu=mvrnorm(1,post_meanbeta,post_sigmabeta)
library(MASS)
postv=solve(priorv+(drop(h02)*xpx))
postbeta=postv%*%(h02*xpy+(priorv%*%beta_mu))
beta=mvrnorm(50,postbeta,postv)

thmean1=thmean+beta
MR=mean(beta[,1])
teta=beta[,1]-MR
lag=sqrt(teta^2)

MR2=mean(beta[,2])
betaa1=mean(beta[,2])
beta1=beta[,2]-betaa1
beeta1=sqrt(beta1^2)
```

```
MR3=mean(beta[,3])
betaa2=mean(beta[,3])
beta2=beta[,3]-betaa2
beeta2=sqrt(beta2^2)

MR4=mean(beta[,4])
betaa3=mean(beta[,4])
beta3=beta[,4]-betaa3
beeta3=sqrt(beta3^2)

sdd=cbind(lag,beeta1,beeta2,beeta3)

thsd1=thsd+sdd
# Posterior mean and standard deviation for theta
cbind(thmean1,thsd1)

lagy=thmean1[,1]
beta0=thmean1[,2]
beta1=thmean1[,3]
beta2=thmean1[,4]

#figure(1)
h=hist(lagy,20, main="Histogram of Posterior Means of the lag of y", xlab = "Lag
y(i)",col="red")
y=density(lagy)
plot(y)


h=hist(lagy,20, main="Histogram of Posterior Means of the lag of y", xlab = "Lag
y(i)",col="red")
xfit<-seq(min(lagy),max(lagy),length=10)
yfit<-dnorm(xfit,mean=mean(lagy),sd=sd(lagy))
yfit <- yfit*diff(h$mids[1:2])*length(lagy)
lines(xfit, yfit, col="blue", lwd=2)
```

```
#figure(2)
g=hist(beta0,20, main="Histogram of Posterior Means of the Beta-(0)", xlab =
"Beta-(0)",,col="red")
k=density(beta0)
plot(k)

g=hist(beta0,20, main="Histogram of Posterior Means of the Beta-(0)", xlab =
"Beta-(0)",,col="red")
xfit<-seq(min(beta0),max(beta0),length=10)
yfit<-dnorm(xfit,mean=mean(beta0),sd=sd(beta0))
yfit <- yfit*diff(g$mids[1:2])*length(beta0)
lines(xfit, yfit, col="blue", lwd=2)

#figure(3)
f=hist(beta1,20, main="Histogram of Posterior Means of the Beta-1", xlab =
"Beta-1(i)",,col="red")
m=density(beta1)
plot(m)

f=hist(beta1,20, main="Histogram of Posterior Means of the Beta-1", xlab =
"Beta-1(i)",,col="red")
xfit<-seq(min(beta1),max(beta1),length=10)
yfit<-dnorm(xfit,mean=mean(beta1),sd=sd(beta1))
yfit <- yfit*diff(f$mids[1:2])*length(beta1)
lines(xfit, yfit, col="blue", lwd=2)

#figure(4)
k=hist(beta2,20, main="Histogram of Posterior Means of the Beta-2", xlab =
"Beta-2(i)",,col="red")
n=density(beta2)
plot(n)
```

```
k=hist(beta2,20, main="Histogram of Posterior Means of the Beta-2", xlab = "Beta-2(i)",,col="red")
xfit<-seq(min(beta2),max(beta2),length=10)
yfit<-dnorm(xfit,mean=mean(beta2),sd=sd(beta2))
yfit <- yfit*diff(k$mids[1:2])*length(beta2)
lines(xfit, yfit, col="blue", lwd=2)
######################################################################
######################################################################
######################################################################
###############################

par(mfrow=c(2,2))
h=hist(lagy,20, main="Histogram of Posterior Means of the lag of y", xlab = "Lag y(i)",col="red")
xfit<-seq(min(lagy),max(lagy),length=10)
yfit<-dnorm(xfit,mean=mean(lagy),sd=sd(lagy))
yfit <- yfit*diff(h$mids[1:2])*length(lagy)
lines(xfit, yfit, col="blue", lwd=2)
g=hist(beta0,20, main="Histogram of Posterior Means of the Beta-(0)", xlab = "Beta-(0)",,col="red")
xfit<-seq(min(beta0),max(beta0),length=10)
yfit<-dnorm(xfit,mean=mean(beta0),sd=sd(beta0))
yfit <- yfit*diff(g$mids[1:2])*length(beta0)
lines(xfit, yfit, col="blue", lwd=2)

f=hist(beta1,20, main="Histogram of Posterior Means of the Beta-1", xlab = "Beta-1(i)",,col="red")
xfit<-seq(min(beta1),max(beta1),length=10)
yfit<-dnorm(xfit,mean=mean(beta1),sd=sd(beta1))
yfit <- yfit*diff(f$mids[1:2])*length(beta1)
lines(xfit, yfit, col="blue", lwd=2)
k=hist(beta2,20, main="Histogram of Posterior Means of the Beta-2", xlab = "Beta-2(i)",,col="red")
xfit<-seq(min(beta2),max(beta2),length=10)
```

```
yfit<-dnorm(xfit,mean=mean(beta2),sd=sd(beta2))
yfit <- yfit*diff(k$mids[1:2])*length(beta2)
lines(xfit, yfit, col="blue", lwd=2)
```

APPENDIX II

Estimation of Parameters and Other Working Graphs

Table 1: The second stage of hierarchical Bayesian Estimation:

$\mu_\gamma / y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ When N=50, T=15, h=25, $\beta_{0i} \sim$ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	β_1	β_2	Precision (h)
Mean	0.01301419	0.01180033	2.01264133	2.30649713	49.772065
Standard deviation	0.00701005	0.02961653	0.02335266	0.02864562	4.12003650
Numerical Standard Error	0.00022167	0.000936556	0.000738476	0.00090585	0.13028699

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} / y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.02134 & -0.00008 & -0.00009 & -0.00002 \\ -0.00008 & 0.02134 & 0.00006 & 0.00017 \\ -0.00009 & 0.00006 & 0.02165 & 0.00006 \\ -0.00002 & 0.00017 & 0.00006 & 0.02126 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00448 & 0.00323 & 0.00330 & 0.00312 \\ 0.00323 & 0.00460 & 0.00331 & 0.00327 \\ 0.00330 & 0.00331 & 0.00467 & 0.00314 \\ 0.00312 & 0.00327 & 0.00314 & 0.00454 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00014 & 0.00010 & 0.00010 & 0.00010 \\ 0.00010 & 0.00015 & 0.00011 & 0.00010 \\ 0.00010 & 0.00010 & 0.00015 & 0.00010 \\ 0.00010 & 0.00010 & 0.00010 & 0.00014 \end{bmatrix}$$

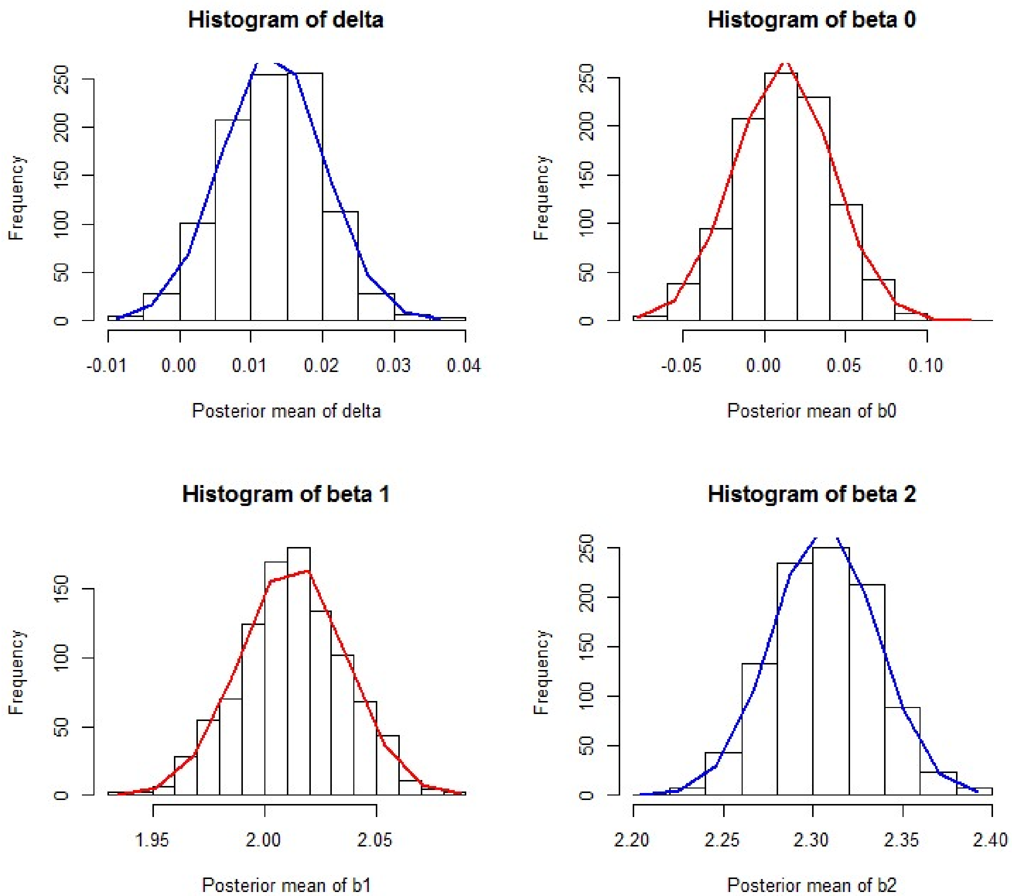

Figure 1: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=50, T=15

Table 2: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=50, T=15, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

Ind.	Posterior Mean				Posterior Standard deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.02768	0.01573	1.98936	2.26469	0.01313	0.00357	0.02042	0.03766
2	-0.00405	-0.01371	2.01589	2.37451	0.01860	0.02588	0.00611	0.07214
3	0.01053	0.04404	1.99659	2.33348	0.00401	0.03187	0.01319	0.03111
4	0.02416	-0.00660	2.03245	2.26791	0.00961	0.01818	0.02267	0.03444
5	0.01240	-0.00811	2.01023	2.33615	0.00214	0.02027	0.00044	0.03379
6	0.01871	0.03910	2.01234	2.25034	0.00416	0.02693	0.00255	0.05201
7	0.01233	-0.04486	2.04893	2.29697	0.00221	0.05703	0.01588	0.00660
8	0.00886	-0.01277	2.04893	2.29697	0.00568	0.02494	0.03914	0.00538
9	0.01078	0.02868	2.00725	2.32655	0.00376	0.01652	0.00254	0.02418
10	0.01540	0.01060	1.99872	2.30867	0.00086	0.00156	0.01107	0.00631
11	0.00855	-0.00229	2.02799	2.30277	0.00599	0.01445	0.01821	0.00040
12	0.02808	-0.01762	1.98166	2.25350	0.01353	0.02979	0.02812	0.04885
13	0.02264	-0.00291	1.96449	2.28837	0.00809	0.01508	0.04529	0.01398
14	0.01244	0.02974	1.99220	2.34018	0.00210	0.01757	0.01758	0.03783
15	0.00122	-0.00330	2.04749	2.34471	0.01332	0.01547	0.03771	0.04235
16	0.03281	0.03714	2.00451	2.21804	0.01826	0.02497	0.00527	0.08432
17	0.01686	0.05813	2.00178	2.29868	0.00232	0.04596	0.00800	0.00367
18	0.01991	0.01233	2.02066	2.29383	0.00535	0.00017	0.01087	0.00853
19	0.01368	-0.00461	1.97803	2.33281	0.00068	0.01677	0.03175	0.03045
20	0.00175	-0.01969	2.07544	2.31209	0.01279	0.03186	0.06565	0.00973
21	0.03104	0.04628	1.96181	2.27269	0.01649	0.03411	0.04797	0.02965
22	0.00618	0.04262	1.99927	2.36356	0.00837	0.03045	0.01051	0.06120
23	0.01632	-0.00721	2.03380	2.26168	0.00177	0.01938	0.02402	0.04067
24	0.01370	0.09373	1.99978	2.30571	0.00084	0.08157	0.01000	0.00335
25	0.00757	0.00181	2.02984	2.25276	0.00697	0.01035	0.02005	0.01078
26	0.02777	-0.01770	1.98967	2.25277	0.01322	0.02987	0.02010	0.04959

27	0.01615	0.00643	2.02886	2.29886	0.00161	0.00573	0.01907	0.00349
28	0.01181	-0.05560	1.97344	2.31777	0.00273	0.06777	0.03633	0.01542
29	0.00291	0.03119	2.04164	2.34060	0.01163	0.01902	0.03185	0.03824
30	0.00611	0.06431	2.01584	2.34143	0.00844	0.05214	0.00605	0.04908
31	0.01335	-0.00469	2.01951	2.29620	0.00119	0.01685	0.00973	0.00616
32	0.02138	0.03549	1.99645	2.27083	0.00683	0.02333	0.01333	0.03152
33	0.01148	-0.00372	2.01294	2.32850	0.00306	0.01588	0.00316	0.02614
34	0.01600	0.02927	2.01537	2.30794	0.00146	0.01710	0.00558	0.00558
35	0.00978	0.02854	2.00145	2.34701	0.00476	0.01637	0.00832	0.04465
36	0.01112	-0.02431	2.01697	2.31987	0.00342	0.03648	0.00719	0.01751
37	0.01369	-0.02748	2.00848	2.29986	0.00085	0.03964	0.00130	0.00249
38	0.02059	-0.01888	1.99114	2.28709	0.00604	0.03105	0.01863	0.01525
39	0.01225	0.05623	2.03054	2.27996	0.00229	0.04406	0.02075	0.02239
40	0.01654	0.02655	1.99284	2.31062	0.00199	0.01438	0.01693	0.00827
41	0.01264	0.04802	1.99901	2.33455	0.00190	0.03586	0.01077	0.03219
42	0.02137	0.00112	1.97604	2.28558	0.00682	0.01104	0.03373	0.01677
43	0.00558	0.02276	2.02052	2.33489	0.00897	0.01059	0.01073	0.03254
44	0.01579	0.03473	1.99057	2.32863	0.00124	0.02257	0.01921	0.02627
45	0.01641	0.00968	2.03510	2.25582	0.00187	0.00248	0.02531	0.04654
46	0.01421	0.00108	2.01184	2.27728	0.00033	0.01108	0.00205	0.01934
47	0.01613	0.04391	1.98737	2.27728	0.00158	0.03175	0.02241	0.02507
48	0.01946	0.01617	2.03138	2.24333	0.00491	0.00400	0.02160	0.05903
49	0.01639	-0.03259	2.01631	2.29771	0.00184	0.04475	0.00653	0.00463
50	0.00863	0.02096	2.02965	2.31090	0.00591	0.00879	0.01986	0.00854

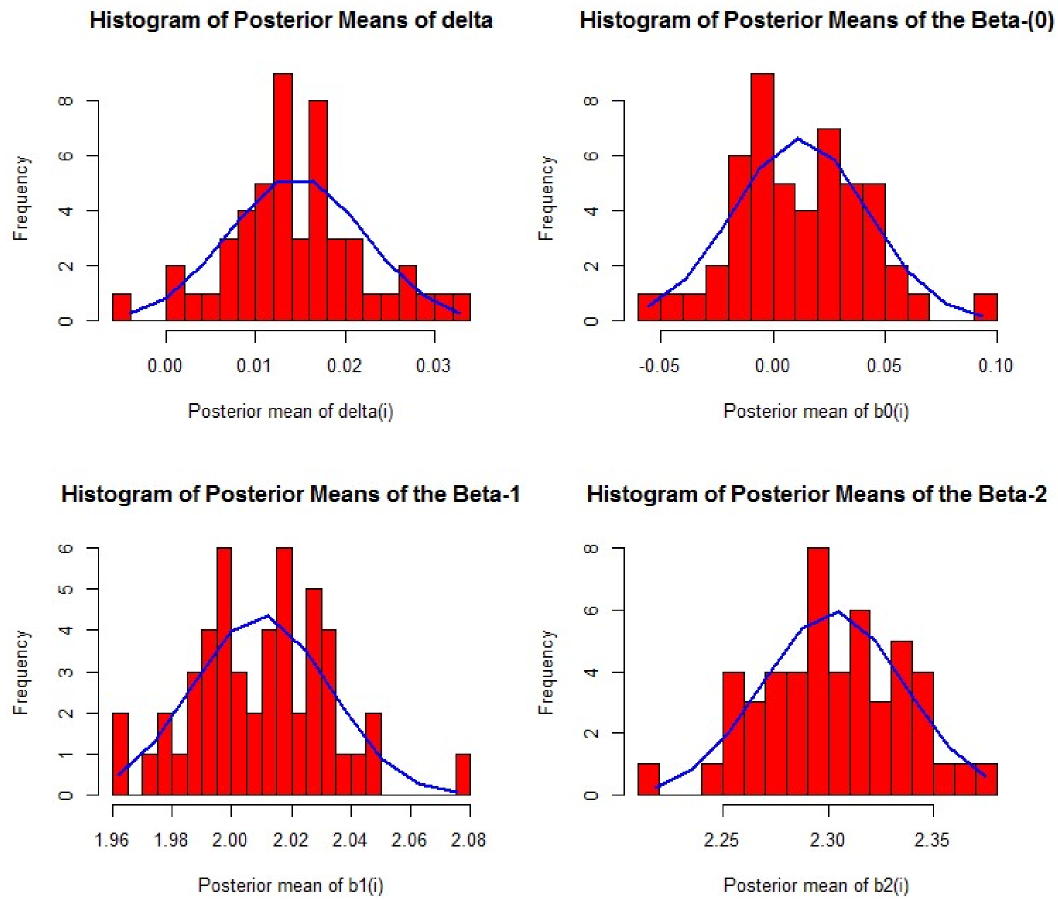

Figure 2: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\overline{\gamma}_i,\ \overline{V}_i)$ for N=50 and T=15

Table 3: The second stage of hierarchical Bayesian Estimation:
$\mu_\gamma / y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ When N=100, T=5, h=25, $\beta_{0i} \sim$ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.08709066	0.01379586	2.19694481	2.33377946	24.12695181
Standard deviation	0.00870379	0.034655746	0.02906209	0.037801703	2.884860961
Numerical Standard Error	0.00027523	0.001095911	0.00091902	0.001195395	0.0912273137

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} / y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.01314 & 0.00003 & -0.00006 & -0.00002 \\ 0.00003 & 0.01284 & -0.00000 & -0.00000 \\ -0.00006 & -0.00000 & 0.01025 & -0.0000 \\ -0.00002 & -0.00000 & -0.00000 & 0.01025 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00144 & 0.00105 & 0.00103 & 0.00099 \\ 0.00105 & 0.00149 & 0.00099 & 0.00106 \\ 0.00099 & 0.00099 & 0.00149 & 0.00101 \\ 0.00099 & 0.00106 & 0.00101 & 0.00151 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00005 & 0.00003 & 0.00003 & 0.00003 \\ 0.00003 & 0.00005 & 0.00003 & 0.00003 \\ 0.00003 & 0.00003 & 0.00005 & 0.00003 \\ 0.00003 & 0.00003 & 0.00003 & 0.00005 \end{bmatrix}$$

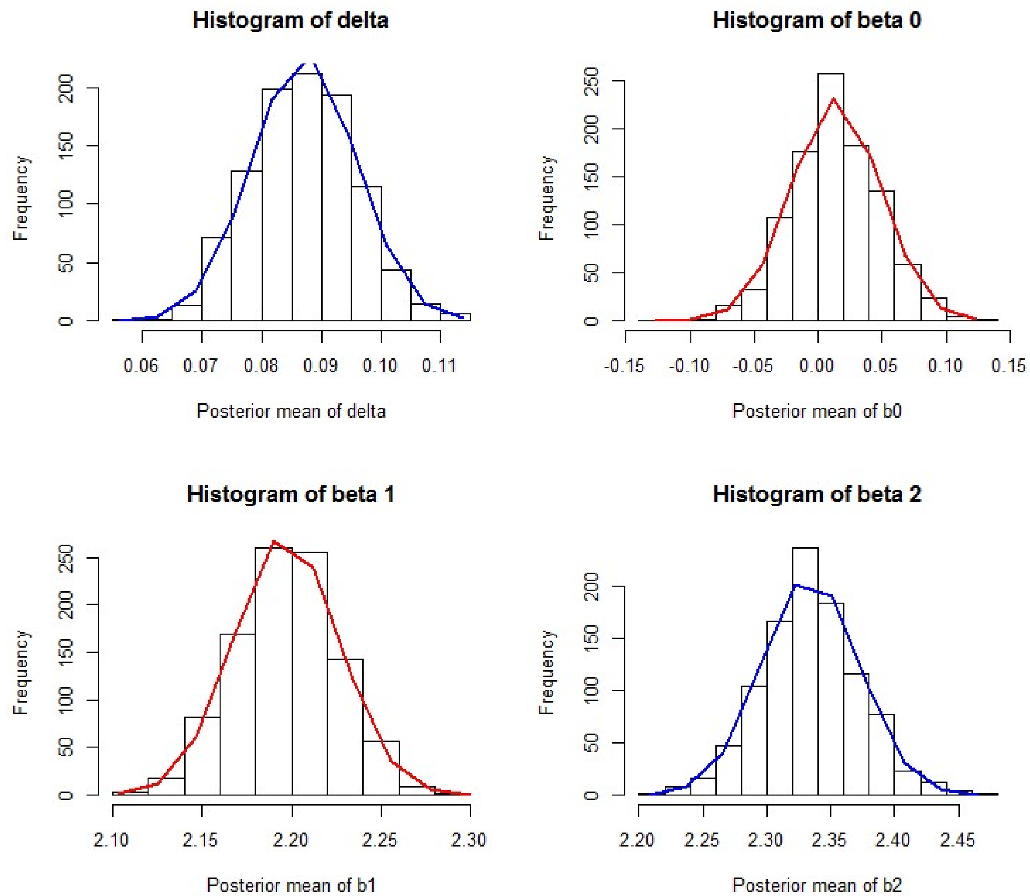

Figure 3: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=100 and T=5

Table 4: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=100, T=5, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior Mean				Posterior Standard deviation			
Ind.	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.08933	0.00231	2.16879	2.32306	0.00889	0.00023	0.21579	0.23114
2	0.08455	0.01429	2.18356	2.34623	0.00841	0.00143	0.21726	0.23344
3	0.09835	0.05824	2.19439	2.28373	0.00979	0.00579	0.21834	0.22723
4	0.09403	0.00280	2.16447	2.33049	0.00936	0.00027	0.21536	0.23188
5	0.09168	0.01358	2.16465	2.31999	0.00912	0.00135	0.21538	0.23084
6	0.08403	0.09682	2.19732	2.34917	0.00836	0.00963	0.21863	0.23374
7	0.07858	-0.02771	2.19480	2.38230	0.00782	0.00276	0.21838	0.23704
8	0.07530	-0.01909	2.22470	2.36888	0.00749	0.00190	0.22136	0.23570
9	0.08586	-0.00981	2.20824	2.35582	0.00854	0.00098	0.21971	0.23440
10	0.08388	0.03923	2.19337	2.35969	0.00835	0.00390	0.21822	0.23477
11	0.09239	-0.03722	2.27031	2.27389	0.00919	0.00370	0.22589	0.22624
12	0.08239	-0.03306	2.23767	2.29818	0.00820	0.00329	0.22265	0.22867
13	0.10338	0.01529	2.13437	2.32099	0.01029	0.00152	0.21237	0.23094
14	0.08023	-0.06855	2.20058	2.36468	0.00798	0.00682	0.21896	0.23528
15	0.09018	0.04411	2.19773	2.32384	0.00897	0.00439	0.21867	0.23122
16	0.08827	-0.02525	2.19857	2.32100	0.00878	0.00251	0.21876	0.23094
17	0.07791	-0.01355	2.20989	2.39319	0.00775	0.00135	0.21988	0.23812
18	0.08963	0.03301	2.21914	2.29930	0.00892	0.00328	0.22080	0.22878
19	0.09012	0.01039	2.19013	2.32415	0.00897	0.00103	0.21791	0.23124
20	0.09030	0.02309	2.19511	2.29861	0.00899	0.00230	0.21841	0.22871
21	0.09679	0.02386	2.17569	2.29998	0.00963	0.00237	0.21647	0.22885
22	0.08177	-0.00038	2.21539	2.34752	0.00814	0.00004	0.22043	0.23357
23	0.09345	0.01103	2.20287	2.31710	0.00929	0.00109	0.21918	0.23055
24	0.10981	-0.01231	2.16079	2.23083	0.01093	0.00123	0.21499	0.22197
25	0.09696	0.06189	2.17820	2.30198	0.00965	0.00616	0.21673	0.22904
26	0.08679	-0.00002	2.22797	2.31497	0.00863	0.00000	0.22168	0.23034
27	0.08533	-0.00575	2.22929	2.32413	0.00849	0.00057	0.22181	0.23125

28	0.09054	0.00704	2.17008	2.32494	0.00901	0.00070	0.21592	0.23133
29	0.08814	0.04554	2.19316	2.33004	0.00877	0.00453	0.21822	0.23184
30	0.09418	0.06919	2.19817	2.31363	0.00937	0.00688	0.21872	0.23024
31	0.09611	0.02585	2.15419	2.31885	0.00956	0.00257	0.21434	0.23072
32	0.08809	-0.05025	2.14267	2.36453	0.00876	0.00500	0.21319	0.23527
33	0.09034	0.01680	2.19486	2.32506	0.00899	0.00167	0.21839	0.23134
34	0.08616	0.01399	2.18396	2.27149	0.00857	0.00139	0.21730	0.23596
35	0.09078	0.05600	2.20266	2.30651	0.00903	0.00557	0.21916	0.22949
36	0.08672	0.01483	2.14223	2.40362	0.00863	0.00147	0.21314	0.23916
37	0.07632	0.06504	2.21589	2.23829	0.00759	0.00647	0.22047	0.23710
38	0.08723	-0.02056	2.17939	2.35542	0.00868	0.00205	0.21685	0.23436
39	0.08681	0.08140	2.18761	2.34695	0.00864	0.00809	0.21766	0.23352
40	0.08946	-0.02504	2.17036	2.32974	0.00890	0.00249	0.21766	0.23181
41	0.07750	-0.06203	2.17170	2.38728	0.00771	0.00617	0.21608	0.23753
42	0.10381	0.01502	2.16987	2.24548	0.01033	0.00149	0.21589	0.22342
43	0.08832	-0.02733	2.20898	2.35615	0.00879	0.00272	0.21979	0.23443
44	0.08852	-0.03050	2.21601	2.31272	0.00881	0.00303	0.22049	0.23011
45	0.08278	-0.01956	2.22730	2.34515	0.00824	0.00195	0.22161	0.23333
46	0.06758	0.05124	2.25552	2.37504	0.00672	0.00600	0.22442	0.23631
47	0.06721	-0.08108	2.24572	2.41108	0.00668	0.00081	0.22345	0.23989
48	0.07793	0.00588	2.25119	2.32858	0.00775	0.00059	0.22399	0.23169
49	0.07104	-0.01923	2.19068	2.42169	0.00707	0.00191	0.21796	0.24095
50	0.10706	-0.02169	2.14632	2.25812	0.01065	0.00215	0.21356	0.22469
51	0.08395	0.01053	2.21453	2.30752	0.00835	0.00105	0.22034	0.22959
52	0.08814	0.00699	2.17273	2.31677	0.00877	0.00069	0.21618	0.23051
53	0.08378	-0.01877	2.23671	2.32985	0.00834	0.00186	0.22255	0.23182
54	0.08340	0.02584	2.24996	2.30694	0.00829	0.00257	0.22387	0.22954
55	0.10084	-0.01243	2.15848	2.29522	0.01003	0.00123	0.21477	0.22837
56	0.07988	0.08443	2.23474	2.32882	0.00794	0.00840	0.22235	0.23171
57	0.08993	0.02206	2.18785	2.32508	0.00895	0.00219	0.21769	0.23134
58	0.08017	0.02390	2.13693	2.39589	0.00798	0.00238	0.21262	0.23839
59	0.09669	0.06535	2.17664	2.29024	0.00962	0.00650	0.21659	0.22788
60	0.09721	-0.00337	2.13702	2.31564	0.00967	0.00034	0.21263	0.23040

61	0.08889	-0.01109	2.16668	2.32903	0.00885	0.00110	0.21558	0.23173
62	0.08239	0.03858	2.20176	2.36247	0.00819	0.00384	0.21907	0.23506
63	0.08470	-0.00706	2.19857	2.30774	0.00842	0.00070	0.21876	0.22962
64	0.08146	0.05560	2.19946	2.38010	0.00810	0.00503	0.21884	0.23682
65	0.08841	-0.01201	2.23071	2.30721	0.00879	0.00119	0.22195	0.22956
66	0.07974	0.00210	2.27156	2.34532	0.00793	0.00021	0.22602	0.23336
67	0.09414	0.00863	2.22826	2.27152	0.00937	0.00086	0.22171	0.22601
68	0.06922	-0.00564	2.23836	2.38629	0.00688	0.00056	0.22271	0.23742
69	0.10496	0.01001	2.14411	2.27571	0.01044	0.00099	0.21334	0.22643
70	0.08982	-0.02489	2.19399	2.32260	0.00894	0.00247	0.21829	0.23109
71	0.07916	-0.04387	2.19222	2.40166	0.00787	0.00437	0.21812	0.23896
72	0.07967	0.02207	2.20272	2.37399	0.00793	0.00219	0.21917	0.23621
73	0.08843	0.00573	2.20298	2.31158	0.00879	0.00057	0.21919	0.22999
74	0.09549	0.03172	2.16486	2.31665	0.00950	0.00316	0.21540	0.23050
75	0.08341	-0.06233	2.22741	2.33588	0.00829	0.00620	0.22162	0.23242
76	0.08335	-0.01319	2.16758	2.37664	0.00829	0.00131	0.21567	0.23647
77	0.07727	0.02004	2.24182	2.35065	0.00769	0.00219	0.22306	0.23387
78	0.07651	0.02958	2.20782	2.36199	0.00761	0.00294	0.21967	0.23502
79	0.08573	0.08301	2.18548	2.33011	0.00853	0.00826	0.21745	0.23184
80	0.09311	0.00754	2.16801	2.33314	0.00926	0.00075	0.21572	0.23215
81	0.07046	0.00187	2.22893	2.43283	0.00701	0.00018	0.22178	0.24206
82	0.08101	-0.01716	2.20139	2.38286	0.00806	0.00107	0.21904	0.23709
83	0.08117	0.01779	2.20944	2.34707	0.00807	0.00177	0.21984	0.23353
84	0.10357	0.01663	2.14334	2.28811	0.01031	0.00165	0.21326	0.22766
85	0.08729	0.00293	2.19675	2.36596	0.00869	0.00029	0.21857	0.23541
86	0.08394	0.06180	2.21431	2.34333	0.00835	0.00615	0.22032	0.23316
87	0.09382	0.00663	2.19051	2.31356	0.09335	0.00066	0.21795	0.23019
88	0.09232	0.00860	2.20472	2.28478	0.00918	0.00085	0.21937	0.22733
89	0.09283	0.02806	2.18218	2.29630	0.00924	0.00279	0.21712	0.22848
90	0.08277	-0.00908	2.18265	2.40919	0.00824	0.00090	0.21717	0.23971
91	0.07295	0.04885	2.23253	2.37111	0.00726	0.00486	0.22231	0.23592
92	0.09063	0.07792	2.20306	2.31776	0.00901	0.00775	0.21920	0.23061
93	0.10118	0.01272	2.18279	2.25345	0.01007	0.00127	0.21718	0.22421

94	0.07551	0.04109	2.22522	2.36423	0.00751	0.00408	0.22141	0.23525
95	0.08999	0.04407	2.18479	2.31561	0.00895	0.00439	0.21738	0.23040
96	0.09091	0.04403	2.17645	2.30147	0.00905	0.00438	0.21655	0.22899
97	0.09327	0.01585	2.19505	2.31035	0.00928	0.00158	0.21841	0.22988
98	0.07040	-0.02254	2.21426	2.39986	0.00701	0.00224	0.22032	0.23878
99	0.10125	0.01878	2.16119	2.30309	0.01007	0.00187	0.21504	0.22916
100	0.11156	-0.07115	2.15376	2.24473	0.01109	0.00708	0.21429	0.22335

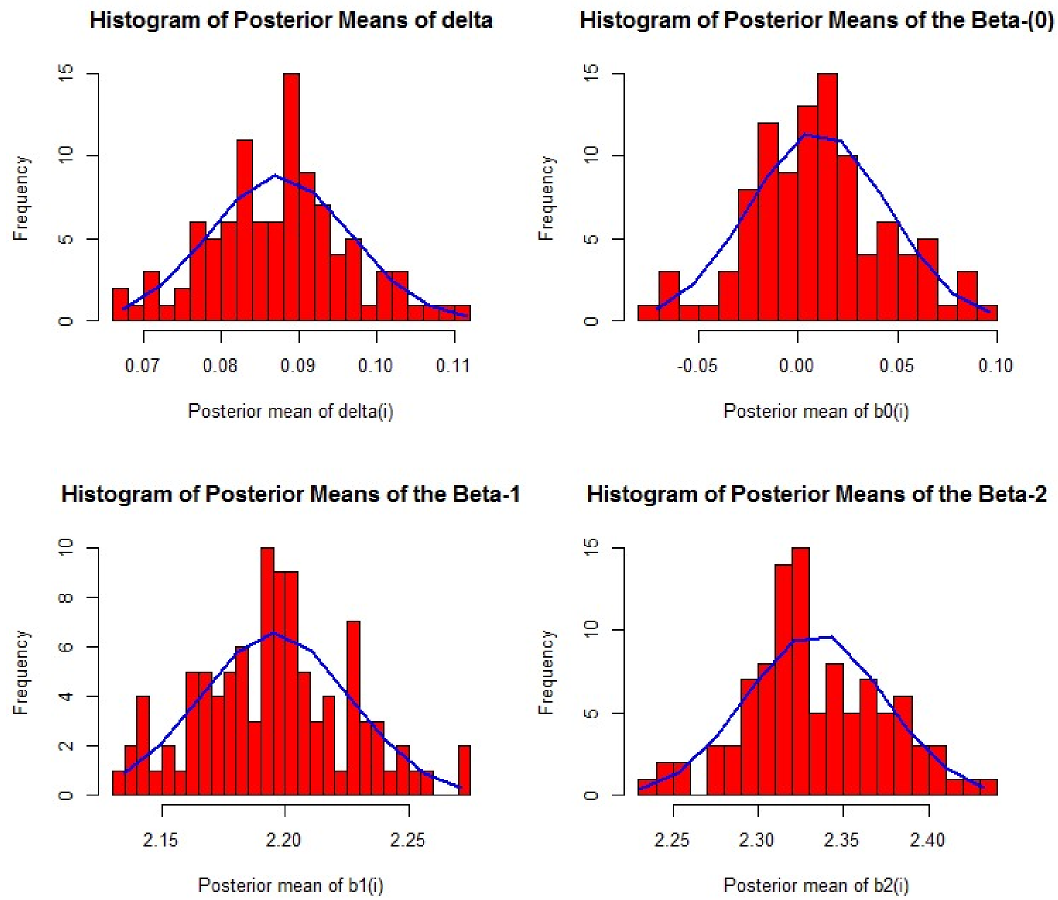

Figure 4: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=100 and T=5

Table 5: The second stage of hierarchical Bayesian Estimation:

$\mu_\gamma / y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ When N=200, T=5, h=25, $\beta_{0i} \sim$ N(0, 0.25) $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.0100976	0.0132563	2.0807478	2.0606310	58.5663764
Standard deviation	0.00603190	0.02387969	0.02173578	0.024998897	4.288568214
Numerical Standard Error	0.0001907	0.0007551	0.00068734	0.0007905345	0.1356164346

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} / y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.00507 & 0.00000 & -0.00000 & -0.00000 \\ 0.00000 & 0.00507 & -0.00000 & -0.00001 \\ -0.00000 & -0.00000 & 0.00507 & -0.00002 \\ -0.00001 & -0.00001 & -0.00002 & 0.00508 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00050 & 0.00036 & 0.00036 & 0.00036 \\ 0.00036 & 0.00052 & 0.00035 & 0.00036 \\ 0.00036 & 0.00035 & 0.00052 & 0.00035 \\ 0.00036 & 0.00036 & 0.00035 & 0.00051 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00002 & 0.00001 & 0.00001 & 0.00001 \\ 0.00001 & 0.00002 & 0.00001 & 0.00001 \\ 0.00001 & 0.00001 & 0.00002 & 0.00001 \\ 0.00001 & 0.00001 & 0.00001 & 0.00002 \end{bmatrix}$$

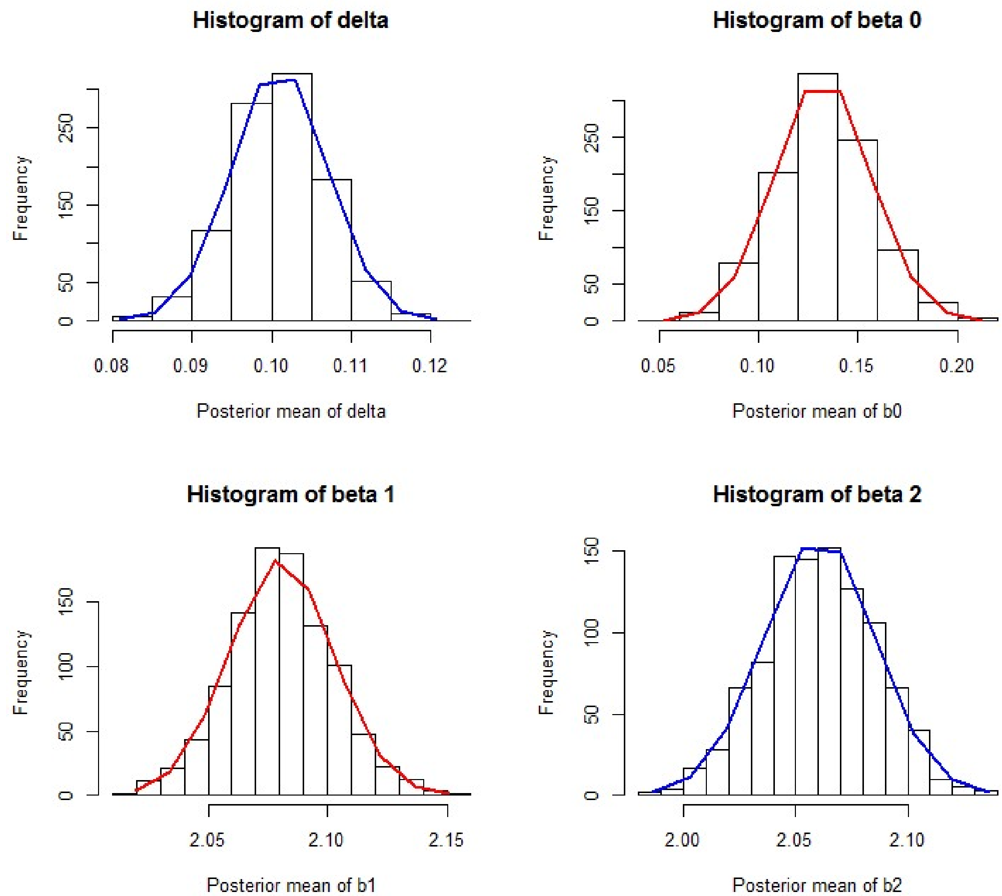

Figure 5: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=200, T=5

Table 6: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=200, T=5 $\beta_{0i} \sim \text{N}(0, 0.25)$, $\delta_i \sim \text{B}(0,1)$

	Posterior Mean				Posterior Standard deviation			
	Posterior standard deviation				Posterior standard deviation			
Ind.	δ	β_0	β_1	β_2	δ	β_0	β_1	β_2
1.	0.10420	0.17477	2.06372	2.05588	0.00320	0.04451	0.01694	0.00523
2.	0.10678	0.09072	2.06415	2.04943	0.00577	0.03954	0.01652	0.01168
3.	0.10167	0.14298	2.09044	2.05509	0.00066	0.01271	0.00976	0.00602
4.	0.10809	0.13923	2.08081	2.08081	0.00201	0.07087	0.00141	0.05077
5.	0.93026	0.09830	2.09059	2.09359	0.00798	0.03196	0.00992	0.03247
6.	0.10496	0.08456	2.07830	2.05491	0.00395	0.04570	0.02364	0.06214
7.	0.10482	0.13365	2.07596	2.05407	0.00381	0.00338	0.00047	0.07054
8.	0.11304	0.15236	2.06197	2.01384	0.00120	0.00220	0.00186	0.00472
9.	0.09251	0.10692	2.08400	2.09922	0.00849	0.02334	0.03331	0.00380
10.	0.09989	0.08875	2.06622	2.06658	0.00111	0.00415	0.01444	0.00546
11.	0.10850	0.13455	2.08164	2.02088	0.00749	0.00428	0.09687	0.04023
12.	0.09998	0.14742	2.08687	2.07432	0.00102	0.00175	0.06202	0.01322
13.	0.09797	0.10111	2.08297	2.07720	0.00303	0.00291	0.00229	0.01607
14.	0.09835	0.10225	2.07127	2.07897	0.00265	0.00280	0.09436	0.01785
15.	0.09833	0.16219	2.08805	2.06805	0.00267	0.03196	0.00737	0.06901
16.	0.10166	0.12323	2.06683	2.05626	0.00065	0.00703	0.01383	0.00486
17.	0.10036	0.11801	2.10256	2.06379	0.00064	0.00122	0.02184	0.00266
18.	0.10393	0.12312	2.07591	2.03610	0.00293	0.00714	0.04760	0.00025
19.	0.09850	0.13806	2.08938	2.04744	0.00241	0.07799	0.08710	0.00136
20.	0.10624	0.16152	2.04901	2.04120	0.00524	0.00315	0.03165	0.01991
21.	0.08713	0.12512	2.11860	2.11295	0.00138	0.05148	0.03793	0.00051
22.	0.09914	0.11444	2.08104	2.08443	0.01864	0.00158	0.00376	0.00023
23.	0.10132	0.14214	2.04582	2.09730	0.00031	0.00118	0.00348	0.03617
24.	0.10612	0.09069	2.05498	2.06038	0.00511	0.00395	0.00256	0.00741
25.	0.99513	0.16497	2.08983	2.05974	0.00149	0.00347	0.09160	0.01337
26.	0.11002	0.11064	2.07141	2.02526	0.00901	0.01962	0.09263	0.00358

27.	0.09870	0.12212	2.09474	2.05271	0.00230	0.00814	0.01406	0.00840
28.	0.09937	0.14495	2.10300	2.03980	0.00163	0.00146	0.00022	0.00213
29.	0.10654	0.07676	2.06516	2.03630	0.00553	0.00535	0.00155	0.00248
30.	0.10713	0.08777	2.06241	2.04173	0.00612	0.00424	0.01826	0.01939
31.	0.09768	0.13453	2.09397	2.05747	0.00332	0.00426	0.00132	0.00365
32.	0.10499	0.11611	2.08422	2.04808	0.00398	0.00141	0.00355	0.00130
33.	0.09645	0.07913	2.11385	2.05554	0.00455	0.00511	0.00331	0.00557
34.	0.11413	0.14189	2.02425	2.03569	0.00131	0.01162	0.00564	0.02542
35.	0.10362	0.15423	2.05555	2.06112	0.00261	0.00239	0.02511	0.03276
36.	0.11384	0.19845	2.05305	2.00147	0.00128	0.06818	0.00276	0.05964
37.	0.09597	0.121352	2.09716	2.07645	0.00503	0.08917	0.01648	0.00153
38.	0.09405	0.123162	2.10735	2.07234	0.00695	0.07107	0.00268	0.00111
39.	0.08877	0.156442	2.10250	2.11659	0.01223	0.02617	0.02183	0.05547
40.	0.10917	0.148524	2.07328	2.02008	0.00816	0.00182	0.00732	0.00410
41.	0.10247	0.138347	2.06624	2.05222	0.00147	0.00807	0.00144	0.00889
42.	0.10120	0.07708	2.09001	2.05971	0.00194	0.00531	0.09342	0.01412
43.	0.10570	0.15412	2.03951	2.06460	0.00469	0.02385	0.04116	0.00347
44.	0.09765	0.13680	2.09562	2.04995	0.00335	0.00653	0.00149	0.01116
45.	0.09839	0.12502	2.10853	2.03983	0.00261	0.00520	0.00279	0.02129
46.	0.09506	0.10530	2.10834	2.08106	0.00059	0.00249	0.02767	0.00199
47.	0.09565	0.11709	2.05723	2.10283	0.00535	0.00131	0.00234	0.04170
48.	0.10882	0.09356	2.05609	2.03539	0.00781	0.00367	0.02457	0.00257
49.	0.10138	0.13330	2.06525	2.06674	0.03761	0.00304	0.01541	0.00056
50.	0.10353	0.13236	2.08719	2.06327	0.00252	0.00209	0.00652	0.00215
51.	0.10888	0.15396	2.07357	2.01046	0.00787	0.02369	0.00710	0.05066
52.	0.10218	0.13191	2.08055	2.05064	0.00117	0.00164	0.00012	0.01047
53.	0.09832	0.12387	2.10255	2.06735	0.00268	0.00639	0.02188	0.00623
54.	0.09303	0.16042	2.09871	2.08814	0.00797	0.03015	0.01804	0.02701
55.	0.09152	0.11831	2.09964	2.09174	0.00948	0.01195	0.01897	0.03062
56.	0.09713	0.14854	2.08998	2.07340	0.00386	0.01827	0.00931	0.01227
57.	0.09534	0.13526	2.11010	2.06345	0.00566	0.00499	0.02943	0.00233
58.	0.09541	0.12564	2.11331	2.06930	0.00559	0.00462	0.03264	0.00818
59.	0.09550	0.12988	2.11347	2.04982	0.00550	0.00038	0.03280	0.01130

60.	0.10239	0.12987	2.06977	2.06141	0.00138	0.00039	0.01089	0.00028
61.	0.10200	0.11918	2.06611	2.05531	0.00100	0.01108	0.01456	0.00580
62.	0.10060	0.11358	2.05835	2.10287	0.00401	0.01668	0.02231	0.04174
63.	0.10055	0.13135	2.11874	2.02481	0.00045	0.00108	0.03806	0.03631
64.	0.10086	0.13856	2.08056	2.05800	0.00013	0.00829	0.00010	0.00312
65.	0.09889	0.12416	2.08828	2.05510	0.00210	0.00610	0.00760	0.00601
66.	0.10108	0.16476	2.08425	2.05352	0.00008	0.03449	0.00358	0.00759
67.	0.09822	0.11766	2.10104	2.05476	0.00278	0.01260	0.02037	0.00635
68.	0.09044	0.13485	2.13471	2.07807	0.01056	0.00458	0.05404	0.01694
69.	0.08990	0.10435	2.11606	2.06519	0.01110	0.02591	0.03539	0.00407
70.	0.09839	0.13004	2.10637	2.06046	0.00261	0.00022	0.02570	0.00061
71.	0.10052	0.11045	2.09337	2.04731	0.00048	0.01981	0.01269	0.01381
72.	0.09621	0.15723	2.07827	2.07068	0.00479	0.02696	0.00239	0.00955
73.	0.10393	0.14876	2.07909	2.04379	0.00292	0.01849	0.00157	0.01733
74.	0.10205	0.10286	2.08518	2.05182	0.00104	0.02740	0.00450	0.00304
75.	0.10744	0.11565	2.04634	2.06618	0.00643	0.01461	0.03432	0.00505
76.	0.09563	0.15237	2.09278	2.06575	0.00537	0.02210	0.01211	0.00462
77.	0.10267	0.15267	2.08026	2.04606	0.00166	0.02240	0.00040	0.01505
78.	0.09911	0.12803	2.04645	2.09120	0.00188	0.00223	0.34215	0.03008
79.	0.10809	0.14458	2.05680	2.06626	0.00708	0.01427	0.02386	0.00513
80.	0.10149	0.09761	2.06608	2.07249	0.00048	0.03265	0.01458	0.01136
81.	0.08911	0.13718	2.09806	2.11105	0.01189	0.00691	0.01733	0.04992
82.	0.09973	0.11024	2.08743	2.07980	0.00127	0.02002	0.00676	0.01868
83.	0.09443	0.14685	2.10443	2.08578	0.00657	0.01659	0.02376	0.02465
84.	0.09452	0.13027	2.10108	2.07985	0.00648	0.00000	0.00204	0.01872
85.	0.10623	0.13885	2.06772	2.05250	0.00522	0.00858	0.01295	0.00862
86.	0.09879	0.08932	2.08640	2.08085	0.00221	0.04094	0.00572	0.01973
87.	0.10292	0.17027	2.07747	2.05078	0.00191	0.04000	0.00319	0.01034
88.	0.09820	0.15830	2.06540	2.09931	0.00280	0.02803	0.01526	0.03818
89.	0.09734	0.12206	2.12718	2.06344	0.00366	0.00820	0.04651	0.00232
90.	0.09739	0.15425	2.10212	2.05682	0.00360	0.02398	0.02145	0.00429
91.	0.09866	0.15425	2.04974	2.10088	0.00234	0.02398	0.03092	0.03975
92.	0.10638	0.07850	2.05691	2.06216	0.00537	0.05176	0.02376	0.00104

93.	0.11047	0.11333	2.06930	2.01693	0.00946	0.01693	0.01137	0.04418
94.	0.09619	0.14312	2.09346	2.09036	0.00480	0.01285	0.01279	0.02924
95.	0.09583	0.13787	2.10520	2.07862	0.00517	0.00760	0.02453	0.01749
96.	0.11122	0.12387	2.05940	2.04113	0.01021	0.00639	0.02127	0.01998
97.	0.10574	0.13043	2.07209	2.04448	0.00473	0.00016	0.00857	0.01663
98.	0.10557	0.10315	2.05573	2.04118	0.00456	0.07117	0.02494	0.01993
99.	0.09638	0.11155	2.09945	2.08054	0.00462	0.01871	0.01878	0.01942
100.	0.10143	0.12958	2.07670	2.07762	0.00043	0.00068	0.00397	0.01650
101.	0.11029	0.09503	2.05567	2.05195	0.00928	0.03523	0.02499	0.00916
102.	0.10108	0.11352	2.06489	2.07356	0.00007	0.01673	0.01577	0.01243
103.	0.10020	0.15394	2.09707	2.05273	0.00080	0.02367	0.01640	0.00839
104.	0.09690	0.12134	2.09438	2.06368	0.00410	0.00892	0.01363	0.00255
105.	0.09520	0.12235	2.09205	2.09831	0.00580	0.00791	0.01138	0.03719
106.	0.09431	0.13299	2.07589	2.09838	0.00669	0.00272	0.00478	0.03725
107.	0.10486	0.13660	2.06943	2.05761	0.00382	0.00633	0.01123	0.00350
108.	0.10209	0.16715	2.05855	2.07715	0.00109	0.03688	0.02212	0.01603
109.	0.09465	0.12721	2.07912	2.09097	0.00635	0.00305	0.00154	0.02985
110.	0.09746	0.16644	2.09137	2.07434	0.00354	0.03617	0.01069	0.01321
111.	0.10040	0.13641	2.06646	2.06201	0.00060	0.00614	0.01420	0.00089
112.	0.09897	0.11172	2.07630	2.07318	0.00202	0.01854	0.00437	0.01205
113.	0.09965	0.12521	2.08239	2.06477	0.00135	0.00505	0.00172	0.00364
114.	0.10809	0.10932	2.04401	2.06695	0.00708	0.02094	0.03665	0.00582
115.	0.10429	0.11716	2.05264	2.06487	0.00328	0.01310	0.02802	0.00374
116.	0.10289	0.09951	2.09244	2.06115	0.00188	0.03075	0.01176	0.00002
117.	0.09279	0.10259	2.10079	2.08702	0.00821	0.02767	0.02012	0.02590
118.	0.09134	0.13280	2.10203	2.08925	0.00966	0.00254	0.02136	0.02813
119.	0.10437	0.13353	2.08549	2.02453	0.00336	0.00326	0.00482	0.03658
120.	0.10284	0.15917	2.04983	2.07929	0.00183	0.02890	0.03083	0.01816
121.	0.09573	0.11588	2.11960	2.06186	0.00527	0.01438	0.03893	0.00074
122.	0.10473	0.14631	2.08985	2.05805	0.00373	0.01604	0.00917	0.00306
123.	0.09773	0.13042	2.09838	2.07047	0.00326	0.00015	0.01771	0.00935
124.	0.09887	0.16170	2.07126	2.07385	0.00213	0.03143	0.00941	0.01273
125.	0.10126	0.12638	2.07331	2.05852	0.00025	0.00388	0.00736	0.00259

126.	0.10470	0.17489	2.06374	2.07300	0.00369	0.04462	0.01693	0.01187
127.	0.10908	0.15859	2.02041	2.05924	0.00807	0.02833	0.06025	0.00187
128.	0.09717	0.14629	2.08155	2.07935	0.00382	0.01602	0.00088	0.01823
129.	0.09923	0.13979	2.12224	2.03078	0.00177	0.00952	0.04157	0.03034
130.	0.09060	0.14022	2.09778	2.11529	0.01039	0.00996	0.01711	0.05417
131.	0.09903	0.13625	2.07590	2.08214	0.00196	0.00598	0.00476	0.02101
132.	0.10043	0.08896	2.07927	2.05346	0.00057	0.04130	0.00139	0.00765
133.	0.11073	0.12799	2.06220	2.02136	0.00972	0.00227	0.01846	0.03975
134.	0.11281	0.13800	2.02565	2.05283	0.01181	0.00773	0.05501	0.00829
135.	0.10488	0.14694	2.07687	2.04006	0.00387	0.01667	0.00379	0.02105
136.	0.10082	0.13845	2.06861	2.04904	0.00018	0.00818	0.01205	0.01207
137.	0.10685	0.17747	2.06992	2.04196	0.00584	0.04720	0.01075	0.01915
138.	0.10321	0.13037	2.11194	2.01416	0.00220	0.00010	0.03127	0.04695
139.	0.10490	0.15997	2.08877	2.03320	0.00390	0.02970	0.00810	0.02792
140.	0.10446	0.12115	2.07442	2.03564	0.00345	0.00911	0.00624	0.02547
141.	0.11101	0.09675	2.05541	2.03682	0.01000	0.03351	0.02525	0.02430
142.	0.10284	0.16882	2.09903	2.02895	0.00183	0.03855	0.01836	0.03217
143.	0.09712	0.11998	2.08535	2.06739	0.00387	0.01028	0.00468	0.00626
144.	0.09917	0.09925	2.08301	2.06868	0.00183	0.03101	0.00234	0.00755
145.	0.10142	0.14166	2.04662	2.07709	0.00041	0.01139	0.03404	0.01596
146.	0.10599	0.09356	2.06579	2.04195	0.00498	0.03670	0.01488	0.01916
147.	0.09678	0.13065	2.07711	2.08306	0.00422	0.00038	0.00355	0.02193
148.	0.10830	0.10537	2.05701	2.03964	0.00729	0.00248	0.02366	0.02147
149.	0.11126	0.17994	2.05439	2.06801	0.01025	0.04967	0.02633	0.05601
150.	0.10360	0.14546	2.05439	2.06801	0.00259	0.01519	0.02628	0.00688
151.	0.10951	0.11818	2.06562	2.03050	0.00850	0.01208	0.01504	0.03061
152.	0.10655	0.07471	2.05704	2.05428	0.00554	0.05555	0.02362	0.00683
153.	0.09042	0.11883	2.09241	2.09953	0.01058	0.01143	0.01174	0.03840
154.	0.09835	0.13656	2.08360	2.08092	0.00265	0.00629	0.00293	0.01980
155.	0.10291	0.06867	2.06999	2.05627	0.00190	0.06159	0.01067	0.00484
156.	0.10009	0.14618	2.10124	2.04332	0.00091	0.01591	0.02056	0.01780
157.	0.10400	0.15083	2.07483	2.04328	0.00299	0.02056	0.00584	0.01783
158.	0.08618	0.14474	2.09453	2.12944	0.01482	0.01447	0.01386	0.06831

159.	0.09394	0.15291	2.09472	2.09732	0.00706	0.02264	0.01405	0.03620
160.	0.09844	0.14447	2.11227	2.04471	0.00256	0.01420	0.03160	0.01641
161.	0.09968	0.09284	2.06207	2.07674	0.00131	0.03742	0.01859	0.01561
162.	0.10159	0.14282	2.07386	2.06290	0.00059	0.01255	0.00681	0.00177
163.	0.10050	0.13045	2.08516	2.06334	0.00050	0.00018	0.00448	0.00221
164.	0.10934	0.13061	2.05103	2.02127	0.00833	0.00034	0.02963	0.03984
165.	0.10104	0.13123	2.08310	2.06267	0.00003	0.00096	0.00242	0.00154
166.	0.09431	0.17086	2.10392	2.07501	0.00668	0.04059	0.02325	0.01388
167.	0.09928	0.11172	2.09167	2.06928	0.00172	0.01854	0.01100	0.00816
168.	0.09931	0.13682	2.06916	2.08025	0.00168	0.00655	0.01150	0.01912
169.	0.10527	0.12159	2.05856	2.04437	0.00427	0.00867	0.02210	0.01675
170.	0.11169	0.11235	2.05481	2.03524	0.01068	0.01791	0.02586	0.02587
171.	0.10276	0.16918	2.07004	2.06629	0.00175	0.03891	0.01062	0.0.5174
172.	0.10170	0.13623	2.08787	2.06439	0.00069	0.00596	0.00719	0.00326
173.	0.11291	0.14592	2.08073	2.01032	0.01190	0.01565	0.00006	0.05080
174.	0.11024	0.18516	2.06067	2.02477	0.00923	0.05489	0.02000	0.03634
175.	0.10621	0.15024	2.07593	2.04414	0.00520	0.01997	0.00474	0.01698
176.	0.10554	0.10760	2.05823	2.06254	0.00454	0.02266	0.02243	0.00141
177.	0.10417	0.15668	2.10192	2.00790	0.00316	0.02641	0.02124	0.05322
178.	0.09943	0.15111	2.07643	2.09184	0.00157	0.02084	0.00424	0.03072
179.	0.09585	0.18204	2.10449	2.07805	0.00515	0.05177	0.02382	0.01692
180.	0.10942	0.13632	2.05873	2.03398	0.00841	0.00605	0.02193	0.02714
181.	0.09824	0.11776	2.09825	2.07954	0.00275	0.01250	0.01758	0.01841
182.	0.11194	0.13841	2.06924	2.02364	0.01093	0.00814	0.01142	0.03747
183.	0.10177	0.17220	2.07643	2.05090	0.00076	0.04193	0.00423	0.01021
184.	0.09682	0.16105	2.08155	2.09183	0.00418	0.03078	0.00087	0.03071
185.	0.08937	0.12461	2.11802	2.06997	0.01163	0.00565	0.03734	0.00885
186.	0.09790	0.09959	2.10924	2.05105	0.00309	0.00306	0.02857	0.01006
187.	0.11030	0.08525	2.06653	2.03512	0.00929	0.04501	0.01413	0.02599
188.	0.08548	0.09248	2.13224	2.10841	0.01551	0.03778	0.05157	0.04728
189.	0.09912	0.08034	2.09495	2.06478	0.00188	0.04992	0.01428	0.00366
190.	0.10149	0.13053	2.08047	2.05029	0.00048	0.00027	0.00020	0.01083
191.	0.10250	0.14554	2.05989	2.06085	0.00149	0.01527	0.02077	0.00026

192.	0.09001	0.17137	2.10949	2.09978	0.01099	0.04110	0.02882	0.00386
193.	0.09497	0.12416	2.10082	2.07600	0.00603	0.00610	0.02014	0.01488
194.	0.09988	0.11564	2.11008	2.06372	0.00111	0.01462	0.02941	0.00260
195.	0.08614	0.09542	2.09415	2.11845	0.01486	0.03484	0.01348	0.05733
196.	0.09802	0.16530	2.08143	2.07966	0.00298	0.03503	0.00076	0.01854
197.	0.10736	0.12548	2.08569	2.01088	0.00635	0.00478	0.00507	0.05024
198.	0.10985	0.09830	2.06031	2.04901	0.00884	0.03196	0.0255	0.01211
199.	0.09966	0.14873	2.08391	2.06113	0.00134	0.01846	0.00324	0.00001
200.	0.10116	0.13186	2.08305	2.05391	0.00015	0.00160	0.00238	0.00720

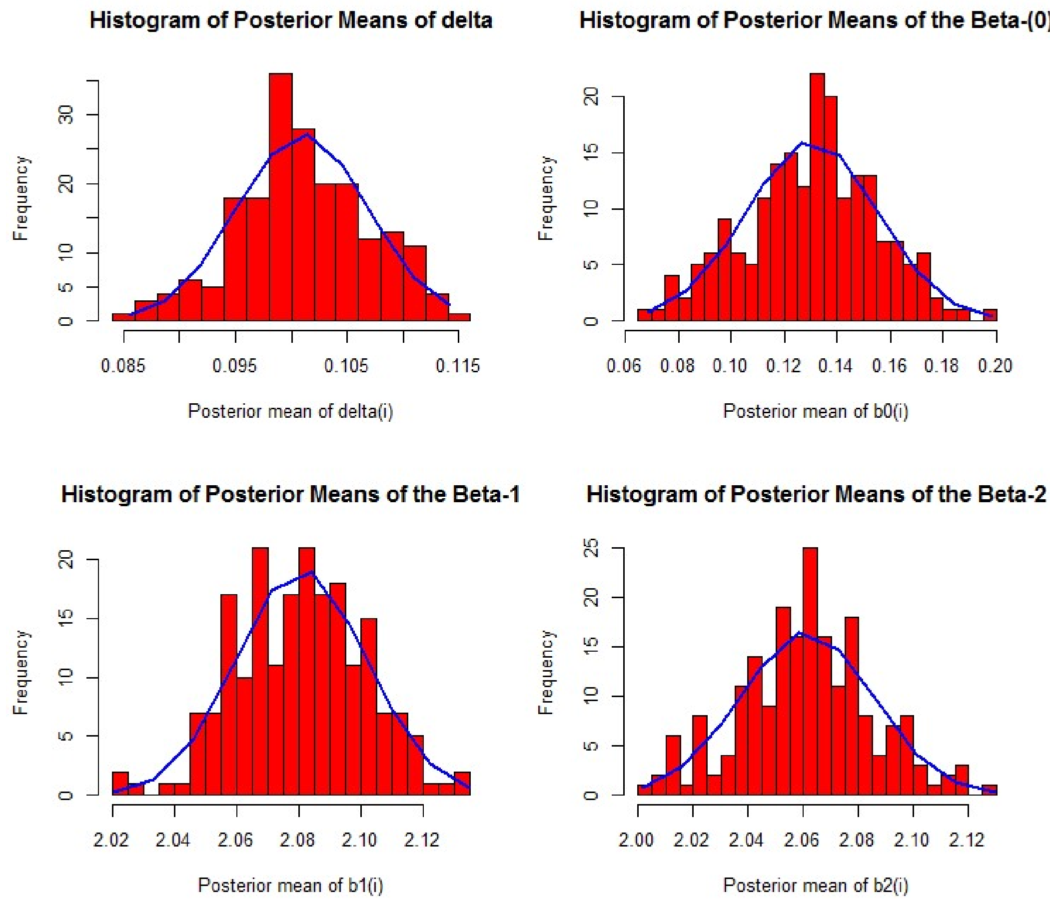

Figure 6: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=200 and T=5

Table 7 The second stage of hierarchical Bayesian Estimation: $\mu_\gamma / y,\gamma,h,V_\gamma \sim N(\overline{\mu}_\gamma,\overline{\Sigma}_\gamma)$

When N=200, T=10, h=25, $\beta_{0i} \sim N(0, 0.25)$ $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.02351085	0.05651572	2.12348264	2.179868	133.4405863
Standard deviation	0.00430891	0.01753861	0.01484233	0.0180112	6.756558950
Numerical Standard Error	0.0001362597	0.0005546198	0.00046935	0.000569	0.21366115

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} / y,\gamma,h,V_\gamma,\mu_\gamma \sim W(\overline{v}_\gamma,\left[\overline{v}_\gamma\overline{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.00509 & 0.00003 & -0.00000 & 0.00001 \\ 0.00003 & 0.00507 & 0.00000 & -0.00001 \\ -0.00000 & 0.00000 & 0.00506 & -0.00001 \\ 0.00001 & -0.00001 & 0.00001 & 0.00506 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00053 & 0.00037 & 0.00038 & 0.00037 \\ 0.00037 & 0.00050 & 0.00036 & 0.00037 \\ 0.00038 & 0.00036 & 0.00051 & 0.00036 \\ 0.00037 & 0.00037 & 0.00036 & 0.00051 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00002 & 0.00001 & 0.00001 & 0.00001 \\ 0.00001 & 0.00002 & 0.00001 & 0.00001 \\ 0.00001 & 0.00001 & 0.00002 & 0.00001 \\ 0.00001 & 0.00001 & 0.00001 & 0.00002 \end{bmatrix}$$

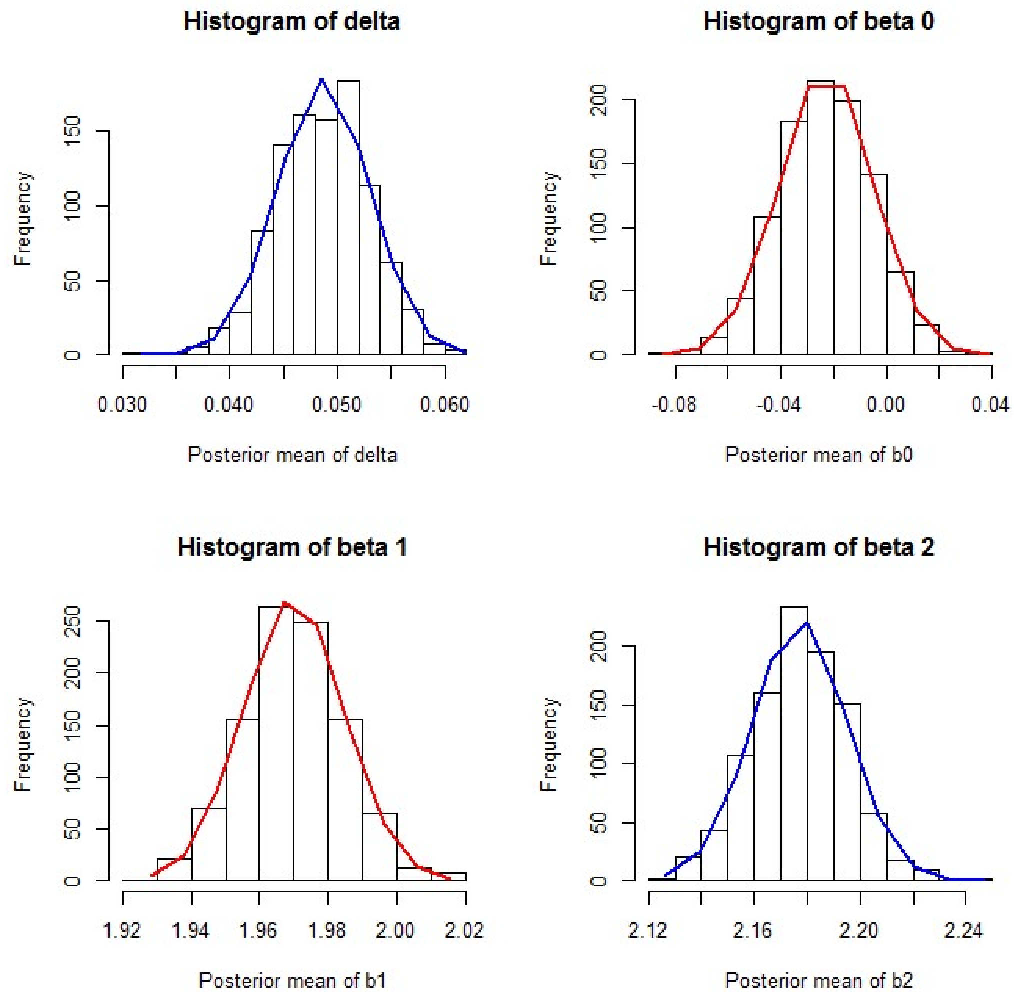

Figure 7: Histograms of posterior means of parameters $\mu_\gamma \mid y, \gamma, h, V_\gamma$ for N=200, T=10

Table 8: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=200, T=10, $\beta_{0i} \sim \mathrm{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior mean				Posterior Standard Deviation			
Ind.	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1.	0.00864	0.05286	2.15850	2.27769	0.01421	0.00371	0.03489	0.04432
2.	0.02122	0.05695	2. 12803	2.17890	0.00163	0.00038	0.00442	0.00440
3.	0.02483	0.05622	2.12213	2.16479	0.00198	0.00035	0.00147	0.01858
4.	0.02070	0.05762	2.13814	2.18397	0.00215	0.00104	0.01454	0.00060
5.	0.02450	0.04689	2.13938	2.16338	0.00164	0.00968	0.01577	0.01998
6.	0.02145	0.05683	2.14245	2.16076	0.00139	0.00025	0.01884	0.02261
7.	0.02106	0.07305	2.11423	2.20678	0.00178	0.01648	0.00937	0.02341
8.	0.02354	0.07175	2.11512	2.18562	0.00068	0.01517	0.00848	0.02246
9.	0.02234	0.04449	2.12392	2.18147	0.00050	0.01208	0.00031	0.00189
10.	0.02344	0.07898	2.12368	2.17681	0.00059	0.02240	0.00007	0.00065
11.	0.02561	0.04474	2.13425	2.16936	0.00276	0.01183	0.01064	0.01401
12.	0.02604	0.05135	2.12935	2.16308	0.00319	0.00521	0.00574	0.02029
13.	0.02463	0.06292	2.11737	2.18651	0.00178	0.00635	0.00623	0.00313
14.	0.02032	0.05372	2.14217	2.17606	0.00252	0.00285	0.01856	0.00731
15.	0.02038	0.08391	2.15966	2.16861	0.00247	0.02734	0.03605	0.01475
16.	0.01794	0.04916	2.12400	2.20165	0.00490	0.00741	0.00004	0.01827
17.	0.01926	0.05436	2.14072	2.17642	0.00359	0.00220	0.01711	0.00695
18.	0.02469	0.07012	2.11507	2.18035	0.00184	0.01354	0.00833	0.00302
19.	0.02365	0.06101	2.13312	2.17691	0.00798	0.00443	0.00951	0.00646
20.	0.02096	0.06077	2.13247	2.19036	0.00189	0.00419	0.00886	0.00698

21.	0.02564	0.05219	2.12540	2.16755	0.00279	0.00437	0.09179	0.01582
22.	0.01905	0.04198	2.14776	2.17285	0.00379	0.01458	0.02415	0.01052
23.	0.02671	0.04616	2.13397	2.14835	0.00386	0.01041	0.01036	0.03501
24.	0.01913	0.05731	2.13984	2.18411	0.00371	0.00074	0.01623	0.00073
25.	0.02658	0.06804	2.11801	2.17521	0.00372	0.01146	0.00558	0.00816
26.	0.02420	0.05896	2.13141	2.17982	0.00134	0.00239	0.00780	0.00355
27.	0.02833	0.03885	2.11424	2.16261	0.00547	0.01772	0.00936	0.02076
28.	0.02962	0.05081	2.11229	2.16323	0.00676	0.00576	0.01131	0.02014
29.	0.02506	0.03972	2.11308	2.18144	0.00221	0.01685	0.01052	0.00193
30.	0.01778	0.02593	2.13496	2.20891	0.00506	0.03063	0.01135	0.02553
31.	0.01707	0.01675	2.13124	2.21075	0.00577	0.03982	0.00763	0.02737
32.	0.02493	0.05495	2.12029	2.17022	0.00208	0.00162	0.00331	0.01314
33.	0.01954	0.07262	2.14262	2.18754	0.00330	0.01605	0.01901	0.00041
34.	0.02145	0.07685	2.13328	2.18322	0.00139	0.02027	0.00967	0.00015
35.	0.01482	0.05028	2.12251	2.21287	0.00802	0.00628	0.00109	0.02950
36.	0.02923	0.05299	2.09812	2.18323	0.00638	0.00357	0.02548	0.00014
37.	0.01959	0.03973	2.12683	2.19267	0.00325	0.01683	0.00322	0.00929
38.	0.01623	0.04619	2.12335	2.22702	0.00661	0.01038	0.00024	0.04364
39.	0.01916	0.07588	2.12420	2.21318	0.00368	0.01930	0.00059	0.02980
40.	0.02667	0.03118	2.09693	2.19176	0.00382	0.02539	0.02666	0.00838
41.	0.02378	0.02818	2.11839	2.18014	0.00093	0.02839	0.00521	0.00323
42.	0.01879	0.05346	2.13458	2.18370	0.00405	0.00311	0.01097	0.00033
43.	0.02012	0.05729	2.13264	2.18141	0.00273	0.00007	0.00903	0.00194
44.	0.02385	0.03164	2.10615	2.18474	0.00100	0.02492	0.01745	0.00137
45.	0.02526	0.03929	2.11286	2.17400	0.00241	0.01727	0.01074	0.00937

46.	0.02862	0.06312	2.11263	2.16212	0.00577	0.00654	0.01097	0.01924
47.	0.02248	0.06084	2.12371	2.19379	0.00036	0.00426	0.00010	0.01041
48.	0.01708	0.04581	2.12672	2.20278	0.00576	0.01076	0.00311	0.01941
49.	0.01972	0.06837	2.11422	2.19764	0.00312	0.01179	0.00938	0.01408
50.	0.02931	0.04484	2.12205	2.17584	0.00646	0.01173	0.00154	0.00753
51.	0.02268	0.07714	2.12457	2.18323	0.00016	0.02056	0.00096	0.00014
52.	0.02694	0.04947	2.11544	2.16850	0.00409	0.00710	0.00816	0.01487
53.	0.02623	0.03810	2.11413	2.17156	0.00338	0.01847	0.00947	0.01181
54.	0.02662	0.00495	2.12402	2.17416	0.00376	0.07008	0.00042	0.00920
55.	0.02388	0.05317	2.14002	2.16119	0.00103	0.00339	0.01641	0.02217
56.	0.02883	0.03638	2.11425	2.18152	0.00597	0.02019	0.00935	0.00185
57.	0.01958	0.03628	2.13411	2.20347	0.00326	0.02029	0.01050	0.02009
58.	0.01773	0.04348	2.14156	2.17939	0.00512	0.01309	0.01795	0.00398
59.	0.02098	0.06862	2.14768	2.16327	0.00186	0.01204	0.02424	0.02010
60.	0.02654	0.01756	2.10494	2.17745	0.00368	0.03901	0.01864	0.00592
61.	0.02653	0.06142	2.12382	2.17296	0.00368	0.00484	0.00021	0.01040
62.	0.02289	0.03313	2.12746	2.18488	0.00438	0.02343	0.00385	0.00150
63.	0.02306	0.05728	2.13022	2.18419	0.00021	0.00070	0.00661	0.00081
64.	0.02027	0.07474	2.11573	2.19612	0.00257	0.01816	0.00787	0.01274
65.	0.01624	0.04408	2.12662	2.19917	0.00661	0.01249	0.00301	0.01579
66.	0.02714	0.04749	2.11019	2.17038	0.00429	0.00908	0.01341	0.01298
67.	0.02098	0.03118	2.11115	2.20039	0.00187	0.02538	0.01245	0.01702
68.	0.02269	0.06140	2.13179	2.18193	0.00015	0.00483	0.00818	0.00144
69.	0.02015	0.05069	2.12871	2.20434	0.00270	0.00587	0.00510	0.02096
70.	0.02255	0.06386	2.13500	2.18398	0.00029	0.00729	0.01140	0.00060

71.	0.02253	0.05056	2.11609	2.19015	0.00031	0.00601	0.00751	0.00677
72.	0.02912	0.05721	2.11255	2.16790	0.00626	0.00063	0.01105	0.01547
73.	0.02043	0.05812	2.13924	2.18055	0.00241	0.00155	0.01563	0.00281
74.	0.01799	0.05051	2.09739	2.21220	0.00485	0.00605	0.02621	0.02882
75.	0.02303	0.05273	2.11273	2.19979	0.00017	0.00383	0.01087	0.01642
76.	0.02531	0.05183	2.11772	2.16824	0.00245	0.00474	0.00588	0.01513
77.	0.02490	0.03783	2.10304	2.18723	0.00205	0.01874	0.02056	0.00385
78.	0.02531	0.07007	2.09723	2.20534	0.00246	0.01350	0.02637	0.02197
79.	0.02599	0.03679	2.11418	2.18167	0.00314	0.01978	0.00942	0.00170
80.	0.02236	0.07398	2.11502	2.18431	0.00048	0.01741	0.00858	0.00093
81.	0.01974	0.06644	2.12139	2.19182	0.00310	0.00986	0.00221	0.00845
82.	0.01925	0.06484	2.11821	2.19983	0.00360	0.00827	0.00539	0.01645
83.	0.02253	0.03895	2.13328	2.17279	0.00031	0.01762	0.00967	0.01058
84.	0.01624	0.04064	2.12491	2.20346	0.00660	0.01593	0.00130	0.02008
85.	0.02213	0.04087	2.08562	2.20597	0.00072	0.01569	0.03798	0.02260
86.	0.02380	0.06576	2.11899	2.19382	0.00094	0.00918	0.00461	0.01044
87.	0.02088	0.05670	2.12865	2.18777	0.00196	0.00012	0.00504	0.00439
88.	0.02211	0.09884	2.13350	2.17114	0.00073	0.04227	0.00989	0.01223
89.	0.01914	0.05267	2.12868	2.19553	0.00370	0.00390	0.00507	0.01215
90.	0.02092	0.08155	2.11414	2.19617	0.00193	0.02498	0.00946	0.01279
91.	0.01422	0.03587	2.16652	2.18219	0.00863	0.02070	0.04291	0.00117
92.	0.02867	0.07345	2.11535	2.14431	0.00581	0.01687	0.00825	0.03906
93.	0.01809	0.05987	2.15429	2.18784	0.00475	0.00329	0.03068	0.00447
94.	0.02936	0.05053	2.11056	2.16464	0.00651	0.00603	0.01304	0.01873
95.	0.02158	0.03835	2.12293	2.19657	0.00127	0.01822	0.00067	0.01320

96.	0.01977	0.09001	2.15632	2.17963	0.00307	0.03343	0.03271	0.00374
97.	0.03071	0.07485	2.11800	2.15570	0.00786	0.01828	0.00560	0.02767
98.	0.02477	0.05395	2.12528	2.17298	0.00192	0.02625	0.00167	0.01038
99.	0.02961	0.08119	2.10736	2.16947	0.00676	0.02461	0.01624	0.01390
100.	0.02049	0.05459	2.14146	2.18183	0.00235	0.00001	0.01785	0.00154
101.	0.02020	0.02833	2.13299	2.18229	0.00264	0.02823	0.00938	0.00108
102.	0.03191	0.07032	2.10861	2.15748	0.00905	0.01374	0.01499	0.02589
103.	0.01681	0.05002	2.12479	2.22418	0.00603	0.00655	0.00118	0.04081
104.	0.01743	0.07162	2.14546	2.18174	0.00542	0.01504	0.02185	0.00163
105.	0.02850	0.05933	2.11699	2.16394	0.00564	0.00276	0.00661	0.01943
106.	0.03436	0.00737	2.08676	2.15765	0.01151	0.01720	0.03684	0.02571
107.	0.02935	0.09744	2.09939	2.16786	0.00650	0.04087	0.02421	0.01550
108.	0.02872	0.08319	2.09870	2.17022	0.00587	0.02662	0.02490	0.01314
109.	0.02099	0.06776	2.13557	2.17343	0.00185	0.01119	0.01196	0.00994
110.	0.02460	0.06338	2.12876	2.18523	0.00175	0.00681	0.00515	0.00186
111.	0.02190	0.05051	2.15263	2.17395	0.00095	0.00605	0.02902	0.00942
112.	0.01835	0.06452	2.14358	2.17583	0.00449	0.00795	0.01997	0.00754
113.	0.01872	0.08782	2.12544	2.20762	0.00412	0.03124	0.00183	0.02425
114.	0.02030	0.08620	2.21177	2.19817	0.00254	0.02963	0.00582	0.01479
115.	0.02246	0.05744	2.09159	2.19893	0.00038	0.00086	0.03201	0.01555
116.	0.02365	0.06781	2.11714	2.17898	0.00079	0.01123	0.00646	0.00439
117.	0.03383	0.05183	2.07931	2.16713	0.01098	0.00474	0.04429	0.01623
118.	0.02574	0.07617	2.12015	2.17602	0.00289	0.01960	0.00345	0.00735
119.	0.02564	0.04248	2.10374	2.19209	0.00278	0.01409	0.01986	0.00872
120.	0.01853	0.05756	2.14277	2.19515	0.00431	0.00098	0.01916	0.01177
121.	0.01416	0.07017	2.13750	2.21181	0.00868	0.01359	0.01389	0.02843
122.	0.01261	0.03896	2.13644	2.21904	0.01023	0.01760	0.01283	0.03567
123.	0.02605	0.05199	2.11336	2.15972	0.00320	0.00045	0.01024	0.02364
124.	0.02787	0.07710	2.10458	2.18029	0.00502	0.00205	0.01902	0.00307

125.	0.01619	0.04092	2.13964	2.20665	0.00665	0.01565	0.01603	0.02327
126.	0.01585	0.05233	2.14312	2.20436	0.00699	0.00424	0.09151	0.02099
127.	0.02315	0.05836	2.13264	2.17656	0.00030	0.00179	0.09034	0.00680
128.	0.02647	0.05001	2.10531	2.18798	0.00362	0.00656	0.01829	0.00460
129.	0.02299	0.02759	2.13524	2.18249	0.00014	0.02897	0.01633	0.00092
130.	0.02437	0.05359	2.09633	2.20434	0.00152	0.00297	0.02727	0.02096
131.	0.02075	0.04945	2.10685	2.20838	0.00209	0.00711	0.01675	0.02500
132.	0.02290	0.06659	2.11452	2.18140	0.00056	0.01001	0.00908	0.00197
133.	0.01980	0.06234	2.14417	2.19007	0.00305	0.00576	0.02056	0.00869
134.	0.02898	0.05362	2.12660	2.14214	0.00613	0.00295	0.00299	0.04123
135.	0.02566	0.04431	2.13014	2.16388	0.00281	0.01226	0.00653	0.01948
136.	0.02060	0.08410	2.13381	2.16982	0.00224	0.02752	0.01020	0.01355
137.	0.01851	0.05595	2.11994	2.20777	0.00433	0.00061	0.00366	0.02439
138.	0.02086	0.04553	2.12041	2.19247	0.00199	0.01104	0.00319	0.00910
139.	0.02163	0.03844	2.12148	2.17595	0.00121	0.01813	0.00212	0.00742
140.	0.02487	0.08119	2.13450	2.15644	0.00202	0.02462	0.01089	0.02693
141.	0.02081	0.05015	2.13360	2.18005	0.00203	0.00642	0.00999	0.00332
142.	0.01827	0.04301	2.13900	2.19260	0.00458	0.01356	0.01539	0.00923
143.	0.02497	0.01245	2.11750	2.17219	0.00211	0.06794	0.00610	0.01117
144.	0.02738	0.05795	2.10097	2.18502	0.00452	0.00138	0.02262	0.00165
145.	0.01820	0.05314	2.15236	2.18306	0.00465	0.00343	0.02876	0.00031
146.	0.02009	0.06525	2.13609	2.18621	0.00275	0.00867	0.01248	0.00283
147.	0.02914	0.06010	2.11226	2.17813	0.00629	0.00352	0.01134	0.00524
148.	0.02311	0.01877	2.12044	2.17355	0.00026	0.03779	0.00136	0.00981
149.	0.01808	0.05727	2.12891	2.19461	0.00476	0.00069	0.00530	0.01123
150.	0.02945	0.04761	2.09522	2.16047	0.00659	0.00895	0.02838	0.02290
151.	0.02456	0.04460	2.12537	2.16318	0.00171	0.01197	0.00176	0.02019
152.	0.02172	0.05399	2.13054	2.18701	0.00112	0.00257	0.00693	0.00364
153.	0.01469	0.04985	2.15279	2.20928	0.00815	0.00672	0.02918	0.02590
154.	0.026463	0.04793	2.11261	2.17021	0.00360	0.00863	0.01099	0.01315
155.	0.016177	0.05884	2.15448	2.18112	0.00668	0.00226	0.03087	0.00225

156.	0.028041	0.06224	2.09989	2.16934	0.00518	0.00567	0.02371	0.01403
157.	0.021017	0.04902	2.13246	2.18447	0.00184	0.00755	0.00886	0.00109
158.	0.022086	0.05615	2.12000	2.20237	0.00077	0.00042	0.00360	0.01899
159.	0.029074	0.03503	2.10541	2.15802	0.00622	0.02153	0.01819	0.02534
160.	0.017996	0.08367	2.12862	2.20662	0.00485	0.02710	0.00502	0.02325
161.	0.024794	0.04360	2.15310	2.14318	0.00194	0.01297	0.02949	0.04019
162.	0.020598	0.06734	2.12514	2.18860	0.00226	0.01076	0.00153	0.00523
163.	0.02344	0.06984	2.10930	2.19229	0.00058	0.01326	0.01430	0.00891
164.	0.01134	0.05525	2.15481	2.20911	0.01151	0.00131	0.03120	0.02573
165.	0.02680	0.06726	2.09758	2.18126	0.00397	0.01069	0.02602	0.00211
166.	0.02447	0.06308	2.10338	2.18854	0.00169	0.00650	0.02022	0.00516
167.	0.02362	0.06431	2.12544	2.16260	0.00077	0.00773	0.01840	0.02076
168.	0.01434	0.07194	2.13669	2.21629	0.00850	0.01536	0.01308	0.03291
169.	0.01950	0.07937	2.13110	2.19877	0.00335	0.02280	0.00749	0.01540
170.	0.02669	0.04635	2.11349	2.18517	0.00384	0.01021	0.01011	0.00179
171.	0.02892	0.08235	2.09709	2.19160	0.00607	0.02577	0.02651	0.00822
172.	0.02584	0.06314	2.11813	2.16998	0.00299	0.00657	0.05474	0.01338
173.	0.02018	-0.0020	2.13482	2.18115	0.00267	0.05861	0.01121	0.00222
174.	0.01804	0.07627	2.13071	2.19448	0.00480	0.01969	0.00710	0.01110
175.	0.02316	0.04520	2.12740	2.15436	0.00031	0.01137	0.00379	0.02900
176.	0.02675	0.05122	2.12101	2.17202	0.00389	0.00535	0.00259	0.01134
177.	0.02646	0.04125	2.10719	2.20700	0.00360	0.01531	0.01641	0.02362
178.	0.01833	0.06014	2.14176	2.20695	0.00451	0.00357	0.01815	0.02357
179.	0.02584	0.07327	2.10969	2.19086	0.00299	0.01669	0.01391	0.00749
180.	0.02550	0.05333	2.11767	2.18988	0.00265	0.00323	0.00593	0.00650
181.	0.03276	0.08785	2.09195	2.16501	0.00990	0.03128	0.03165	0.01835
182.	0.01584	0.08178	2.14066	2.21542	0.00700	0.02511	0.01705	0.03205
183.	0.02676	0.05613	2.10815	2.18450	0.00390	0.00044	0.01545	0.00113
184.	0.02504	0.05388	2.12415	2.18434	0.00218	0.02695	0.00054	0.00096
185.	0.03238	0.06588	2.08602	2.15989	0.00953	0.00931	0.03758	0.02348
186.	0.02549	0.04439	2.14598	2.15532	0.00264	0.01218	0.02238	0.02804

187.	0.02555	0.07672	2.13199	2.16480	0.00270	0.02014	0.00838	0.01856
188.	0.02808	0.05653	2.10896	2.18121	0.00522	0.00003	0.01464	0.00216
189.	0.02450	0.03749	2.11527	2.17750	0.00165	0.01907	0.00833	0.00586
190.	0.01987	0.08411	2.13121	2.20759	0.00366	0.02753	0.00760	0.02421
191.	0.02289	0.07038	2.11483	2.17173	0.00143	0.01380	0.00877	0.01163
192.	0.02809	0.06210	2.12661	2.16016	0.00523	0.00552	0.00301	0.02320
193.	0.02039	0.07103	2.14428	2.18238	0.00246	0.01445	0.02067	0.00099
194.	0.02389	0.06171	2.11615	2.18594	0.00104	0.00514	0.00745	0.00256
195.	0.02038	0.03698	2.12787	2.19295	0.00246	0.01959	0.00427	0.00958
196.	0.02657	0.06768	2.10991	2.16846	0.00372	0.01104	0.01369	0.01491
197.	0.02467	0.05276	2.12858	2.19087	0.00181	0.00380	0.00497	0.00749
198.	0.02119	0.04529	2.11572	2.19921	0.00166	0.01128	0.00787	0.01583
199.	0.02130	0.03274	2.12013	2.20098	0.00154	0.02383	0.00347	0.01760
200.	0.02262	0.03470	2.12964	2.16614	0.00022	0.02186	0.00603	0.01723

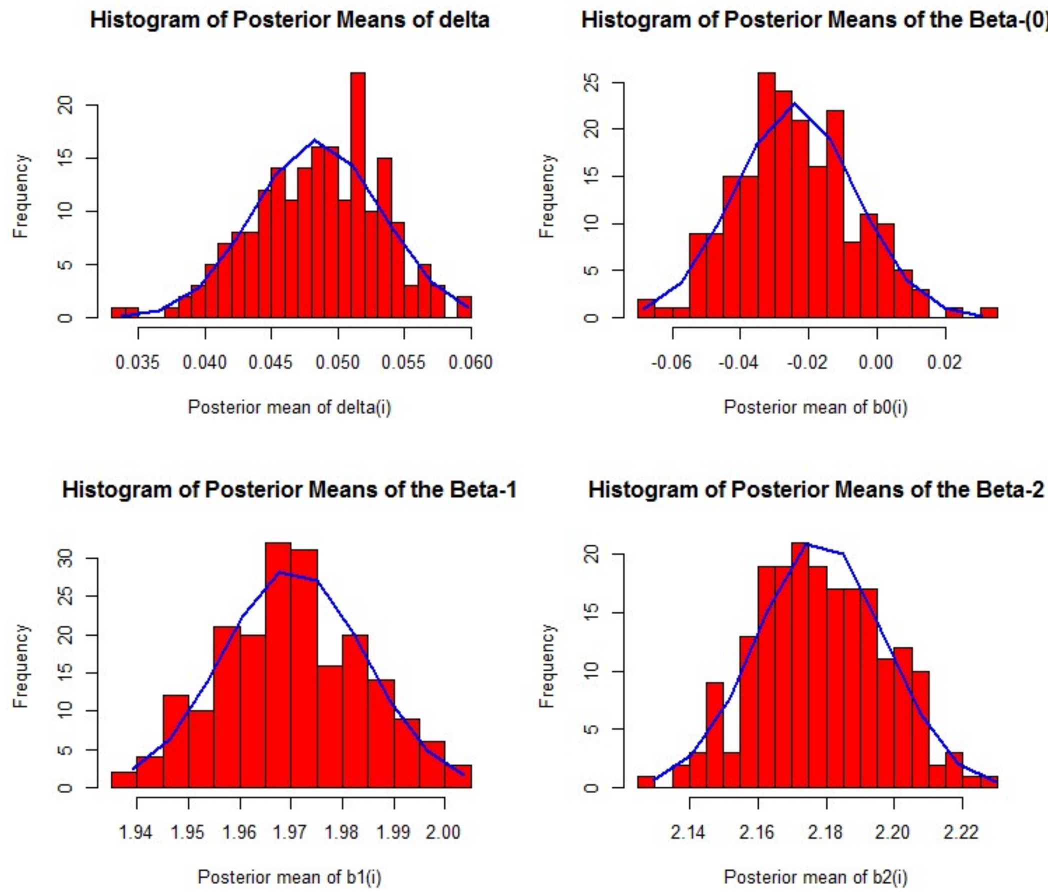

Figure 8: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=200 and T=10

Table 9: The second stage of hierarchical Bayesian Estimation:

$\mu_\gamma / y, \gamma, h, V_\gamma \sim N(\overline{\mu}_\gamma, \overline{\Sigma}_\gamma)$ When N=200, T=15, h=25, $\beta_{0i} \sim N(0, 0.25)$ $\delta_i \sim B(0,1)$

	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	Precision (h)
Mean	0.12762977	0.02278219	2.07172861	1.96920079	188.20921407
Standard deviation	0.003484823	0.01429229	0.01264774	0.014603830	7.877382659
Numerical Standard Error	0.000110199	0.00045196	0.00039995	0.000461814	0.249104712

The posterior estimates of variance covariance matrix for V_γ :

$$V_\gamma^{-1} / y, \gamma, h, V_\gamma, \mu_\gamma \sim W(\overline{v}_\gamma, \left[\overline{v}_\gamma \overline{V}_\gamma\right]^{-1})$$

Mean

$$\begin{bmatrix} 0.00506 & 0.00000 & -0.00000 & -0.00000 \\ 0.00000 & 0.00506 & -0.00000 & 0.00000 \\ -0.00000 & -0.00000 & 0.00506 & -0.00002 \\ -0.00001 & 0.00000 & -0.00002 & 0.00507 \end{bmatrix}$$

Standard deviation

$$\begin{bmatrix} 0.00050 & 0.00036 & 0.00036 & 0.00036 \\ 0.00036 & 0.00052 & 0.00035 & 0.00036 \\ 0.00036 & 0.00035 & 0.00051 & 0.00035 \\ 0.00036 & 0.00036 & 0.00035 & 0.00050 \end{bmatrix}$$

Numerical Standard Error

$$\begin{bmatrix} 0.00002 & 0.00001 & 0.00001 & 0.00001 \\ 0.00001 & 0.00002 & 0.00001 & 0.00001 \\ 0.00001 & 0.00001 & 0.00002 & 0.00001 \\ 0.00001 & 0.00001 & 0.00001 & 0.00002 \end{bmatrix}$$

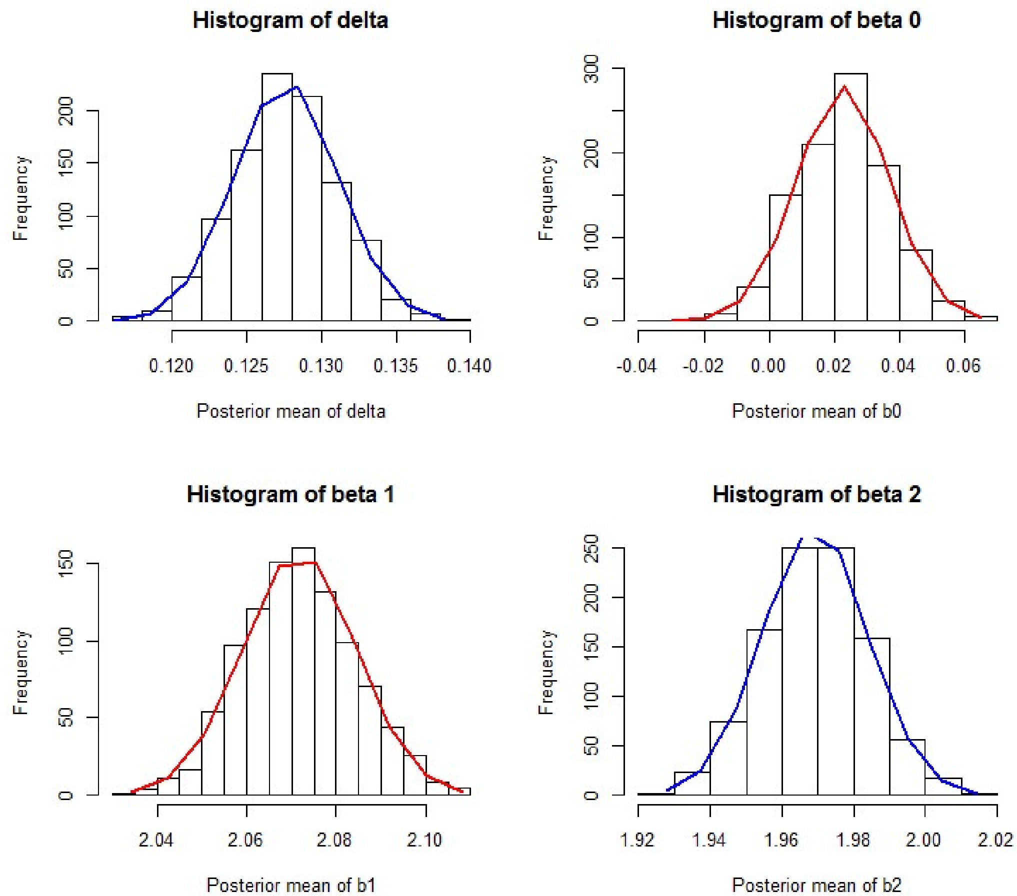

Figure 9: Histograms of posterior means of parameters $\mu_{\gamma} \mid y, \gamma, h, V_{\gamma}$ for N=200, T=15

Table 10: Posterior mean and Posterior Standard deviation for the first stage of hierarchical Bayesian Estimates: $\gamma_i / y_i, h, \mu_\gamma, V_\gamma \sim N(\overline{\gamma}_i, \overline{V}_i)$ When N=200, T=15, $\beta_{0i} \sim \text{N}(0, 0.25)$ $\delta_i \sim B(0,1)$

	Posterior Mean				Posterior Standard Deviation			
Ind.	δ	β0	$\beta_1(2)$	$\beta_2(3)$	Δ	β0	$\beta_1(2)$	$\beta_2(3)$
1	0.12664	0.02063	2.07747	1.96528	0.00162	0.00017	0.00793	0.00280
2	0.13151	0.03941	2.05431	1.96944	0.00324	0.01860	0.01522	0.00135
3	0.12076	0.02073	2.09496	1.98969	0.00751	0.00007	0.02541	0.02160
4	0.12739	0.00711	2.08454	1.96156	0.00087	0.01369	0.01499	0.00652
5	0.12866	0.026359	2.06392	1.97134	0.00038	0.05546	0.00562	0.00325
6	0.12679	0.01215	2.08543	1.96936	0.00147	0.03296	0.01589	0.00127
7	0.12884	0.01486	2.07987	1.95382	0.00056	0.00595	0.01033	0.01426
8	0.12744	0.00499	2.06320	1.97724	0.00082	0.01582	0.00633	0.00915
9	0.12690	0.02209	2.08132	1.96322	0.00136	0.00128	0.01178	0.00486
10	0.12735	0.01465	2.08248	1.96116	0.00091	0.00615	0.01294	0.00692
11	0.12991	0.02545	2.06405	1.96002	0.00164	0.00464	0.00548	0.00806
12	0.12496	0.04345	2.06577	1.98130	0.00330	0.02264	0.00377	0.01321
13	0.13262	0.03563	2.05652	1.94791	0.00434	0.01482	0.01302	0.02017
14	0.12754	0.02048	2.07125	1.97256	0.00072	0.00033	0.00170	0.00447
15	0.12788	0.01331	2.06189	1.97401	0.00039	0.00749	0.00765	0.00592
16	0.12928	0.02030	2.06677	1.97396	0.00100	0.00050	0.00276	0.00587
17	0.12645	0.00108	2.04446	2.00832	0.00181	0.02189	0.02507	0.04023
18	0.12276	0.00457	2.08454	1.97895	0.00551	0.01623	0.01500	0.01086
19	0.12664	0.01255	2.05422	1.98966	0.00162	0.00825	0.01531	0.02157
20	0.12558	0.00774	2.05822	1.98777	0.00268	0.01306	0.11318	0.01968
21	0.13309	0.02950	2.04762	1.95652	0.00482	0.00869	0.02191	0.01156
22	0.12188	0.01997	2.07358	1.99075	0.00639	0.00084	0.00403	0.02266
23	0.12278	0.01306	2.08596	1.99415	0.00549	0.00775	0.01642	0.02606
24	0.13564	0.01569	2.04453	1.95010	0.00737	0.00511	0.02501	0.01798
25	0.12726	0.00131	2.07626	1.96276	0.00100	0.02213	0.00672	0.00532
26	0.13005	0.02602	2.05224	1.97403	0.00178	0.00520	0.01729	0.00594
27	0.12900	0.02162	2.08275	1.95854	0.00073	0.00081	0.01321	0.00955

28	0.12919	0.01805	2.08277	1.94527	0.00091	0..00274	0.01323	0.02281
29	0.12937	0.01083	2.07151	1.96527	0.00110	0.00991	0.00197	0.00281
30	0.13383	0.01529	2.06111	1.94259	0.00556	0.00551	0.00843	0.02549
31	0.12908	0.00799	2.05862	1.97004	0.00081	0.02881	0.01091	0.00195
32	0.13596	0.04959	2.06265	1.93748	0.00768	0.02878	0.00689	0.03060
33	0.12300	0.00304	2.07760	1.98757	0.00526	0.02385	0.00805	0.01948
34	0.12742	0.04052	2.07277	1.96596	0.00084	0.01970	0.00322	0.00212
35	0.12624	0.02744	2.09453	1.96643	0.00202	0.00663	0.02499	0.00165
36	0.13092	0.02064	2.04431	1.97698	0.00265	0.04146	0.02523	0.00889
37	0.12524	0.02404	2.07434	1.97721	0.00302	0.00323	0.00480	0.00912
38	0.13556	0.0189	2.05787	1.94030	0.00729	0.00189	0.01167	0.02778
39	0.12253	0.04347	2.07628	1.98940	0.00574	0.02266	0.00674	0.02131
40	0.13000	0.04191	2.07913	1.96086	0.00173	0.02118	0.00958	0.00727
41	0.12801	0.00936	2.07351	1.96605	0.00025	0.01145	0.00397	0.00206
42	0.12159	0.04748	2.08969	1.98390	0.00667	0.02665	0.02014	0.01588
43	0.13333	0.01217	2.06105	1.95721	0.00506	0.00869	0.00848	0.01083
44	0.12576	0.03832	2.07058	1.97861	0.00250	0.01751	0.00100	0.01058
45	0.12346	0.02466	2.06926	1.99249	0.00481	0.00385	0.00020	0.02440
46	0.12992	0.02929	2.05651	1.97276	0.00165	0.00848	0.01306	0.00467
47	0.12136	0.01438	2.07922	1.98365	0.00690	0.00643	0.00966	0.01556
48	0.12685	0.01631	2.06457	1.97911	0.00142	0.00449	0.00491	0.01102
49	0.12303	0.01590	2.09786	1.97453	0.00523	0.00491	0.02835	0.00644
50	0.12415	0.04042	2.07280	1.98415	0.00412	0.01961	0.00323	0.01606
51	0.13058	0.02466	2.06491	1.96226	0.00231	0.00329	0.00464	0.00582
52	0.13583	0.01302	2.06694	1.93568	0.00756	0.00778	0.00259	0.03240
53	0.13582	0.01446	2.05163	1.95112	0.00754	0.00634	0.01796	0.01696
54	0.13090	0.01935	2.06128	1.95399	0.00263	0.00145	0.00825	0.01409
55	0.12854	0.01182	2.06182	1.96867	0.00027	0.00898	0.00770	0.00058
56	0.13027	0.01939	2.08186	1.94606	0.00199	0.00142	0.01232	0.02202
57	0.12269	0.03774	2.07522	1.98320	0.00557	0.01693	0.00563	0.01511
58	0.12412	0.03659	2.06330	1.99651	0.00415	0.01578	0.00620	0.02842
59	0.12801	0.04125	2.06571	1.96928	0.00025	0.02044	0.00383	0.00119
60	0.13246	0.01966	2.05928	1.95292	0.00418	0.001152	0.01021	0.01516

61	0.12610	0.03193	2.09726	1.95734	0.00216	0.01112	0.02777	0.01074
62	0.12462	0.01015	2.07515	1.98006	0.00364	0.01065	0.00563	0.01197
63	0.12405	0.03216	2.08444	1.98375	0.00422	0.01134	0.01487	0.01566
64	0.12450	0.00424	2.06604	2.00409	0.00376	0.01656	0.00349	0.03600
65	0.12885	0.01483	2.07886	1.95943	0.00058	0.00597	0.00934	0.08653
66	0.12789	0.00017	2.08879	1.95171	0.00037	0.02098	0.01929	0.01637
67	0.13054	0.00869	2.05103	1.97297	0.00226	0.01211	0.01851	0.00488
68	0.13373	0.02868	2.06771	1.95491	0.00546	0.00787	0.00180	0.01317
69	0.13401	0.01100	2.07398	1.94162	0.00574	0.00980	0.00444	0.02646
70	0.12832	0.04466	2.06339	1.97984	0.00048	0.02385	0.00615	0.01175
71	0.12928	0.03224	2.06530	1.97063	0.00101	0.01143	0.00421	0.00254
72	0.12648	0.05196	2.07468	1.97700	0.00178	0.03114	0.00513	0.00891
73	0.12345	0.00942	2.08403	1.98074	0.00482	0.01138	0.01443	0.01265
74	0.13450	0.02611	2.04986	1.94995	0.00623	0.00529	0.00197	0.01813
75	0.12152	0.01739	2.08577	1.98245	0.00674	0.00341	0.01627	0.01436
76	0.13053	0.00234	2.06481	1.94695	0.00226	0.01846	0.00470	0.02113
77	0.13042	0.01516	2.07952	1.95241	2.14850	0.00565	0.00993	0.01567
8	0.12116	0.01970	2.07850	1.99673	0.00710	0.00110	0.00895	0.02864
79	0.12511	0.01209	2.06312	1.98004	0.00316	0.00871	0.00641	0.01195
80	0.13210	0.02390	2.04404	1.96474	0.00383	0.00309	0.02549	0.00334
81	0.12587	0.00606	2.09316	1.96726	0.00239	0.01475	0.02362	0.00082
82	0.13087	0.00666	2.06190	1.95366	0.00259	0.01414	0.00764	0.04428
83	0.12698	0.02987	2.06542	1.97708	0.00128	0.00905	0.00412	0.00899
84	0.12435	0.00884	2.07144	1.99038	0.00392	0.01196	0.00189	0.02229
85	0.13415	0.02472	2.06349	1.94905	0.00588	0.00391	0.00605	0.01903
86	0.12754	0.01251	2.06621	1.96780	0.00072	0.00829	0.00333	0.00028
87	0.12615	0.02093	2.07781	1.97487	0.00211	0.00012	0.00826	0.00678
88	0.12929	0.02691	2.07193	1.96451	0.00101	0.00610	0.00239	0.00357
89	0.12402	0.01986	2.05619	1.99565	0.00424	0.00094	0.01335	0.02756
90	0.12656	0.02885	2.07378	1.97458	0.00170	0.00804	0.00423	0.00649
91	0.13453	0.0015	2.07027	1.92731	0.00626	0.01923	0.00073	0.04077
92	0.13029	0.05225	2.05640	1.95012	0.00202	0.03147	0.01313	0.01796
93	0.13599	0.02239	2.07352	1.92407	0.00772	0.00156	0.00397	0.04401

94	0.12684	0.01958	2.06532	1.97923	0.00143	0.00123	0.00421	0.01114
95	0.12885	0.01372	2.07573	1.94746	0.00057	0.00703	0.00619	0.02062
96	0.12716	0.00190	2.06196	1.97450	0.00110	0.01882	0.00758	0.00641
97	0.12532	0.01283	2.08338	1.97125	0.00294	0.00800	0.01383	0.00316
98	0.13233	0.03610	2.04661	1.96140	0.00405	0.01529	0.02293	0.0066
99	0.13213	0.02155	2.05533	1.95527	0.00386	0.00077	0.01421	0.01285
100	0.12105	0.01937	2.09288	1.99391	0.00722	0.00142	0.02334	0.02584
101	0.12741	0.02732	2.07689	1.96203	0.00085	0.00651	0.00735	0.00602
102	0.12391	0.01502	2.08091	1.98095	0.00435	0.00576	0.01137	0.01283
103	0.13180	0.01505	2.06959	1.95446	0.00352	0.00578	0.00050	0.01362
104	0.13061	0.03712	2.06790	1.95932	0.00234	0.01638	0.00164	0.00873
105	0.12898	0.04709	2.05643	1.97090	0.00070	0.00262	0.01311	0.00289
106	0.12465	0.01040	2.05962	1.99513	0.00362	0.01040	0.00991	0.02707
107	0.13454	0.02551	2.04298	1.95827	0.00626	0.00470	0.02656	0.00981
108	0.12360	0.03309	2.08964	1.97155	0.00467	0.01227	0.02009	0.00346
109	0.13375	0.03377	2.05955	1.94987	0.00548	0.01295	0.00999	0.01821
110	0.12451	0.00495	2.10044	1.97278	0.00375	0.01586	0.03089	0.00469
111	0.13102	0.04760	2.05663	1.95635	0.00275	0.02679	0.01290	0.01173
112	0.13283	0.02712	2.06431	1.96099	0.00456	0.00631	0.00522	0.00709
113	0.12864	0.03612	2.07259	1.96715	0.00036	0.01531	0.00469	0.00093
114	0.13001	0.02360	2.05478	1.97632	0.00174	0.00279	0.01476	0.00823
115	0.12686	0.01404	2.08175	1.97299	0.00140	0.00677	0.01221	0.00490
116	0.12841	0.03278	2.06328	1.97978	0.00013	0.01197	0.00626	0.01169
117	0.12751	0.00215	2.07732	1.97654	0.00075	0.02296	0.00777	0.00845
118	0.12799	0.01371	2.05153	1.99098	0.00027	0.00709	0.01801	0.02289
119	0.13077	0.01448	2.06930	1.95600	0.00249	0.00632	0.00023	0.01208
120	0.13803	0.01967	2.04050	1.95757	0.00976	0.00113	0.02903	0.01051
121	0.12388	0.00734	2.10592	1.95512	0.00438	0.02816	0.03637	0.01296
122	0.13524	0.03286	2.04770	1.96489	0.00696	0.01205	0.02184	0.00319
123	0.13159	0.03867	2.05750	1.95451	0.00332	0.01786	0.01203	0.01357
124	0.13127	0.03958	2.06111	1.96189	0.00300	0.01877	0.00842	0.00619
125	0.12845	0.03433	2.06816	1.96353	0.00017	0.01352	0.00137	0.00455
126	0.12513	0.00880	2.07973	1.97250	0.00313	0.01200	0.01019	0.00441

127	0.12698	0.01062	2.05287	1.97944	0.00129	0.01018	0.01666	0.01135
128	0.12904	0.01682	2.06801	1.95771	0.00077	0.00399	0.00152	0.01037
129	0.12679	0.00290	2.08011	1.95532	0.00147	0.01791	0.01057	0.00127
130	0.13366	0.02088	2.06063	1.95567	0.00538	0.00007	0.00890	0.01241
131	0.12999	0.00072	2.05759	1.97318	0.00172	0.02008	0.01194	0.00509
132	0.13137	0.02118	2.07394	1.94951	0.00310	0.00037	0.00439	0.01857
133	0.12439	0.02410	2.09669	1.96134	0.00387	0.00329	0.02714	0.00674
134	0.12269	0.02879	2.08780	1.97387	0.00557	0.00798	0.01825	0.00578
135	0.13165	0.02908	2.05410	1.96744	0.00337	0.00827	0.01544	0.00064
136	0.12659	0.01438	2.07489	1.96487	0.00167	0.00642	0.00534	0.00321
137	0.13195	0.02109	2.06154	1.95815	0.00368	0.00027	0.00799	0.00993
138	0.13015	0.01092	2.06656	1.96133	0.00188	0.00988	0.00297	0.00675
139	0.12681	0.00923	2.06371	1.96801	0.00145	0.01157	0.00583	0.00007
140	0.12825	0.01051	2.05781	1.97324	0.00013	0.01029	0.01173	0.00515
141	0.12924	0.03348	2.07549	1.95658	0.00097	0.01267	0.00594	0.01150
142	0.13068	0.00748	2.08399	1.93110	0.00240	0.01332	0.01445	0.03698
143	0.13327	0.01766	2.03928	1.95363	0.00499	0.00314	0.03025	0.01445
144	0.13193	0.00416	2.05760	1.96101	0.00366	0.01665	0.01194	0.00707
145	0.12401	0.01193	2.08014	1.98643	0.00425	0.00887	0.01060	0.01834
146	0.12544	0.02880	2.09146	1.97319	0.00282	0.00799	0.00219	0.00510
147	0.12611	0.05948	2.06820	1.97500	0.00216	0.03866	0.00133	0.00691
148	0.13109	0.00727	2.06845	1.96225	0.00281	0.01353	0.00109	0.00583
149	0.12916	0.03471	2.05184	1.98255	0.00089	0.01390	0.01770	0.01446
150	0.12900	0.01539	2.06777	1.96125	0.00072	0.00542	0.00177	0.00683
151	0.12439	0.03870	2.08048	1.98485	0.00387	0.01789	0.01094	0.01676
152,	0.12046	0.02142	2.08191	1.99656	0.00780	0.00060	0.01236	0.02847
153	0.13553	0.03069	2.04375	1.95738	0.00726	0.00987	0.02579	0.01070
154	0.12420	0.01630	2.06360	1.99175	0.00406	0.00450	0.00594	0.02366
155	0.12745	0.00038	2.06063	1.97567	0.00081	0.02042	0.00891	0.00758
156	0.12622	0.02978	2.05849	1.97030	0.00205	0.00897	0.01105	0.00221
157	0.12817	0.01588	2.05511	1.98280	0.00096	0.00496	0.01443	0.01471
158	0.12853	0.03488	2.08371	1.95784	0.00026	0.01406	0.01416	0.01024
159	0.12832	0.02274	2.09403	1.94160	0.00056	0.00192	0.02449	0.02648

160	0.12963	0.02199	2.07084	1.98181	0.00135	0.00117	0.00130	0.01372
161	0.12986	0.01386	2.07796	1.97322	0.00158	0.00697	0.00842	0.00513
162	0.12385	0.01273	2.08226	1.98589	0.00441	0.03354	0.01272	0.01780
163	0.11992	0.02468	2.07938	2.00646	0.00834	0.00383	0.00984	0.03837
164	0.12818	0.00677	2.08336	1.96776	0.00087	0.01403	0.01381	0.00032
165	0.12870	0.02520	2.08301	1.94665	0.00043	0.00439	0.01346	0.02143
166	0.12623	0.03283	2.06216	1.97704	0.00204	0.01202	0.00737	0.00895
167	0.13252	0.02036	2.06069	1.94672	0.00425	0.00045	0.00884	0.02136
168	0.12322	0.00506	2.08061	1.99715	0.00505	0.01575	0.01107	0.02906
169	0.12539	0.02769	2.06748	1.97514	0.00288	0.00687	0.00205	0.00705
170	0.12683	0.00857	2.06905	1.98255	0.00144	0.01223	0.00049	0.01446
171	0.12988	0.01523	2.06073	1.95993	0.00161	0.00558	0.00880	0.00815
172	0.12521	0.03396	2.08185	1.97095	0.00305	0.01315	0.01231	0.00286
173	0.12732	0.01413	2.06809	1.97358	0.00095	0.00668	0.00145	0.00549
174	0.12870	0.03988	2.04954	1.99231	0.00042	0.01906	0.01999	0.02422
175	0.13322	0.01413	2.05964	1.94975	0.00495	0.00667	0.00990	0.01833
176	0.13141	0.00334	2.08002	1.95030	0.00313	0.01746	0.01048	0.01778
177	0.12998	0.01131	2.06232	1.97480	0.00171	0.00949	0.00721	0.00671
178	0.12554	0.02336	2.09254	1.96265	0.00273	0.00255	0.02300	0.00543
179	0.12973	0.01520	2.06567	1.96446	0.00146	0.00560	0.00387	0.00362
180	0.13028	0.01588	2.06114	1.96916	0.00200	0.00492	0.00840	0.00107
181	0.12713	0.02258	2.06739	1.97353	0.00114	0.00177	0.00215	0.00544
182	0.12130	0.03387	2.09605	1.98180	0.00696	0.01306	0.02651	0.01371
183	0.13035	0.03530	2.05212	1.97411	0.00208	0.01448	0.01741	0.00602
184	0.12818	0.02089	2.08333	1.95577	0.00090	0.00086	0.01379	0.01231
185	0.12778	0.01393	2.07386	1.95248	0.00212	0.00535	0.00432	0.01201
186	0.13040	0.02616	2.07049	1.95248	0.00221	0.00553	0.00095	0.01560
187	0.13207	0.01971	2.06729	1.93837	0.00380	0.00109	0.00225	0.02971
188	0.12417	0.04078	2.07512	1.98037	0.00409	0.01997	0.00558	0.01228
189	0.12935	0.014814	2.05750	1.96504	0.00108	0.00599	0.01203	0.00304
190	0.13113	0.03090	2.06303	1.95077	0.00286	0.01009	0.00650	0.01731
191	0.13617	0.02227	2.07180	1.92201	0.00790	0.00146	0.00226	0.04607
192	0.12996	0.03670	2.05919	1.96566	0.00168	0.01588	0.01034	0.00242

193	0.12812	0.02552	2.06573	1.97610	0.00014	0.00471	0.00380	0.00801
194	0.12706	0.00127	2.06238	1.98301	0.00121	0.01954	0.00716	0.01492
195	0.12554	0.03465	2.06996	1.98359	0.00272	0.01383	0.00041	0.01550
196	0.13352	0.01775	2.05920	1.96750	0.00525	0.00305	0.01033	0.00058
197	0.12362	0.02917	2.08099	1.98254	0.00464	0.00835	0.01145	0.01445
198	0.12963	0.04200	2.06776	1.95531	0.00136	0.02119	0.00178	0.01277
199	0.12652	0.04082	2.07750	1.97984	0.00175	0.02000	0.00796	0.01175
200	0.12311	0.02357	2.09296	1.97400	0.00515	0.00276	0.02342	0.00591

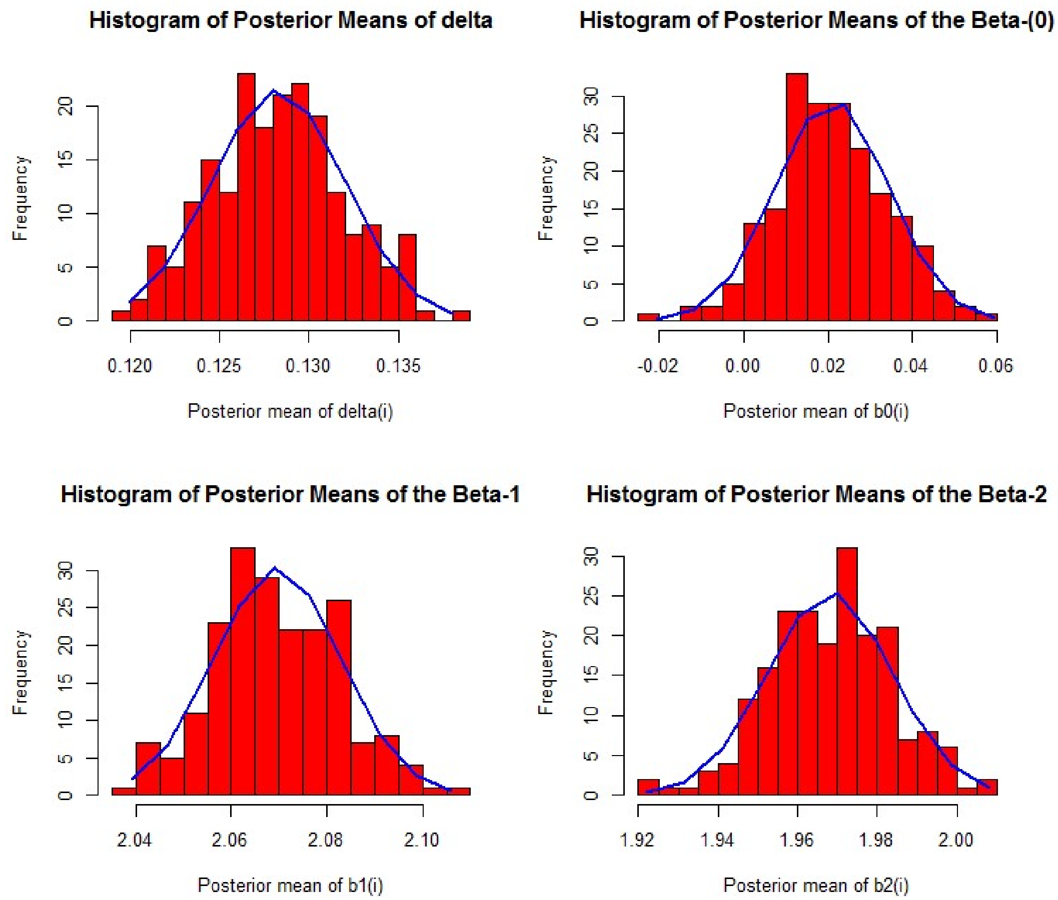

Figure 10: Histogram of posterior mean of parameters $\gamma_i \mid y, h, \mu_\gamma, V_\gamma \sim \mathrm{N}(\bar{\gamma}_i,\ \bar{V}_i)$ for N=200 and T=15